VAGUENESS AND TRUTH

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Summary: It seems to be a foregone conclusion that natural language meanings are vague. Much depends, however, on the way how vagueness is analyzed. For example, should vagueness be treated in terms of the truth- or denotation-conditions of expressions? Rather than proposing yet another "fuzzy" or multi-valued logic, the present paper investigates the nature of *reference* and *truth*. We consider various types of formal models proposed in the literature of logical semantics and discuss what they are suited as well as unsuited to treat. Then we turn to the ontology of these models, i.e. the question of what the set-theoretic structures called a model are supposed to stand for intuitively. Two main possibilities are contrasted. One is called the paradigm I approach, according to which the model-strucutre is interpreted as a representation of reality (such that the speaker/hearer is part of the model structure). The other is called the paradigm II approach, according to which the model structure is interpreted as a representation of conceptual meaning structures (such that the model structure is part of the speaker/hearer). It is shown that the theoretical nature of vagueness is totally different in the two paradigms. In conclusion, a number of standard examples of vagueness are analyzed within the paradigm II approach, including the so-called Sorites paradox or paradox of the heap.

Zusammenfassung: Die Annahme erscheint unvermeidlich, daß Bedeutungen in natürlichen Sprachen vage sind. Vieles hängt jedoch davon ab, wie Vagheit analysiert wird. Sollte sie z.B. in Form von Wahrheits- oder Denotationsbedingungen behandelt werden? In der vorliegenden Arbeit wird nicht eine weitere mehrwertige oder "Fuzzy"-Logik vorgeschlagen, sondern vielmehr das Wesen von *Referenz* und *Wahrheit* untersucht. Wir betrachten verschiedene Typen formaler Modelle, die in der Literatur der Logischen Semantik vorgeschlagen wurden, und erörtern, welche Probleme sich damit behandeln lassen und welche nicht. Dann wenden wir uns der Ontologie dieser Modelle zu, d.h. der Frage, wofür die als Modell bezeichneten mengentheoretischen Strukturen unseren intuitiven Annahmen gemäß stehen. Zwei hauptsächliche Möglichkeiten werden gegenübergestellt: Der einen gemäß (die wir als Paradigma-I-Ansatz bezeichnen) wird die Modellstruktur als Repräsentation der Wirklichkeit interpretiert (so daß der Sprecher/Hörer Teil der Modellstruktur ist), gemäß der anderen (genannt Paradigma-II-Ansatz) wird die Modellstruktur als Repräsentation begrifflicher Bedeutungsstrukturen interpretiert (so daß die Modellstruktur Teil des Sprecher/Hörers ist). Es wird gezeigt, daß die theoretische Beschaffenheit der Vagheit in den beiden Paradigmen völlig verschieden ist. Abschließend wird eine Reihe von Standardbeispielen für Vagheit innerhalb des Paradigma-II-Ansatzes analysiert, darunter das sogenannte Sorites- oder Haufen-Paradox.

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It is often claimed that one difference between natural languages, like English or German, and formal languages, like predicate calculus or intensional logic, is that the former are inherently vague and inconsistent, whereas the latter are precise and consistent. This view is not limited to logicians who regard formal languages as a means to escape what they perceive as the pitfalls and irregularities of natural languages, but may also be found among linguists whose primary concern is the analysis of natural languages. LAKOFF

(1972: 183), for example, claims that "natural language concepts have vague boundaries and fuzzy edges and that, consequently, natural language sentences will very often be neither true, nor false, nor nonsensical, but rather true to a certain extent and false to a certain extent, true in certain respects and false in other respects".

What are the consequences of this widely accepted view? If sentences are not true or false simpliciter, but true or false to a certain degree, then the traditional two-valued logic systems do not suffice, but must be extended into many-valued logics. And indeed, when we look at different proposals to treat vagueness, such as LAKOFF (1972), KAMP (1975), BLAU (1977), PINKAL (1981), KINDT (1983), and others, we find the premises inherent in Lakoff's formulation quoted above are accepted. The concern of these authors is the construction of different multi-valued systems. These multi-valued logics differ insofar as they borrow motivation and/or formal proposals from different other areas, such as probability or measurement theory à la Kolmogorov (Kamp), mathematical topology (Kindt), supervaluations (Pinkal, Kamp), fuzzy logic à la Zadeh (Lakoff), or three-valued logic in the tradition of Lukasiewicz (Blau).

But what is the common premise underlying these formal approaches to vagueness? And does it adequately capture the intuitive nature of vagueness? Let us illustrate the common premise underlying the above mentioned proposals with a few examples: When we observe the process of slowly closing the door, then, we are told, this raises the question at what point the sentence 'The door is open' is still true and at what point the sentence is false. One may even feel impelled to ask to what *degree* the sentence is true or false at various stages of closing the door. And similarly for the sentence 'The door is closed'.

Another situation in which logicians and linguists have found vagueness is the classification of colors. If an object is called *red* in some context, but non-*red* in another context, doesn't it follow that the natural language concept of *red* is vague? Indeed, if we consider applying the predicates 'x is red' and 'x is orange' to the transition from red to orange on a color spectrum, the problem is similar to the first example.

The same considerations may be applied in the evaluation of an adjective like *big*. How much bigger than the average fly must Xerxes be in order for the sentence 'Xerxes is a big fly' to be true? Note, however, that the question of degrees of truth and the related question of vagueness of certain words must be clearly distinguished from certain other issues frequently brought into the discussion, namely the intensionality of certain adjectives. The fact that 'Xerxes is a big fly' does not entail 'Xerxes is a big entity' has nothing whatsoever to do with the vagueness of *big*. After all, there are also vague predicates like *red* which are completely extensional. Thus 'Xerxes has red eyes' clearly entails 'Xerxes' eyes are red entities'. And conversely, there are adjectives like *alleged* or *fake* which are intensional but not vague.

The present paper deals solely with the intuitive nature of vagueness and the proper formal implementation of vagueness within model-theoretic semantics. As such it will be concerned with the nature of *reference*. The syntactico/semantic treatment of comparatives (e.g. 'x is bigger than y') the distinction between intensional and extensional predicates, and other questions of this kind will be left to other occasions.

Our treatment of vagueness proceeds from different basic assumptions than the aforementioned approaches in that for us vagueness is essentially a pragmatic phenomenon. We will show that the construal of semantic vagueness in the above examples is an absurd artefact of a misguided ontological interpretation of model-theoretic semantics. For us, neither the literal meaning of 'The door is open' nor of 'This stone is red' is vague. Vagueness does not arise in the literal meaning concepts of natural language concepts (pace Lakoff), but rather in the pragmatic process of reference, which we define as the matching relation between the sharply defined concepts of natural language meanings (so-called icons) and the contextual objects to which these icons refer. Thus we propose to treat vagueness in terms of the pragmatic notion of language use (reference) rather than the semantic notion of truth- or denotation-conditions.

Our analysis of vagueness differs from the traditional treatments within formal logic in that it does not add yet another multi-valued system to those already in existence. This is not because we agree with certain conservative logicians who want to retain traditional two-valued systems and/or see no use in the logical analysis of natural language meaning. On the contrary, we believe firmly in the model-theoretic analysis of natural language meaning within the general framework of Montague grammar. Furthermore, we have been using a non-bivalent logic (namely a presuppositional intensional logic based on partially defined functions and logical connectives defined à la Kleene) in order to describe so-called P-induced semantic presuppositions (HAUSSER, 1976).

But whereas semantic presuppositions are a denotation conditional property of natural surface expressions, vagueness is not. For this reason it is mistaken to treat presupposition failure and vagueness in terms of the same formal system, i.e. a semantics based on multi-valued logic. The origin of this mistake is the failure of traditional model-theoretic systems to distinguish between semantics and pragmatics. Semantics deals with the truth- (or rather denotation-) conditional analysis of the literal meaning of natural language expressions. Pragmatics, on the other hand, analyzes the use of natural language expressions by a speaker/hearer relative to a context.

In order to give our alternative approach to vagueness a precise characterization within model theory, we consider in section 1 different possible interpretations of existing model-theoretic systems. One possibility is treating the model structure as a representation of reality, with the consequence that the speaker/hearer(s) are part of the model structure. This approach is the presently most widely accepted interpretation of the model structure and will be called the paradigm I approach. The other possibility is treating the model structure as something conceptual, with the consequence that the model structure is part of the speaker/hearer (formalized as a speaker simulation device or SID). This second ontological interpretation of the model structure has been advocated in HAUSSER (1980b, 1981a, b, 1982) and will be called the paradigm II approach.

One important advantage of the paradigm II approach is the clear separation of the semantic and the pragmatic interpretation of a token. The semantic interpretation consists (roughly speaking, cf. section 3 for details) in the SID-internal construction of a model that makes the sentence true (the so-called token model). The pragmatic interpretation, on the other hand, consists in matching the token model with the so-called context model, defined as what the SID perceives and remembers at the moment of interpretation. It is this particular set up of the paradigm II approach that provides the basis for our alternative treatment of vagueness.

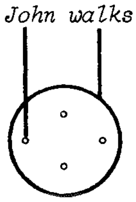
After describing the basic features of the paradigm I and the paradigm II approach in section 1, we turn in section 2 to a comparison of the two paradigms, especially with regards to the respective treatment of truth and vagueness. In section 3 we show that presupposition failure and vagueness are completely different phenomena in a paradigm II system. Section 4 discusses the treatment of several examples of vagueness in a paradigm II system and proposes a solution to the so-called Sorites paradox.

1. Formal model theory and its ontological interpretation

What is the nature and the function of a formal model in logic? Logic originated as a theory of deduction. The goal was and is to derive valid conclusions from given premises. Thereby, two types of sentences are distinguished. Those which are true (tautologies) or false (contradictions) solely on the basis of their syntactic structure. And those whose truth-value depends on the "situation" to which they refer (contingent sentences). For example, the sentence 'John walks or John doesn't walk' is always true because of its tautological structure, but the sentence 'John walks' depends for its truth-value on the situation under consideration.

Model-theoretic semantics (in the tradition of WITTGENSTEIN, 1922, CARNAP, 1947, KRIPKE, 1963, and MONTAGUE, 1974) provides a formal (set-theoretic) description of the situations relative to which contingent sentences may be interpreted. We say 'John walks' is 1 (true) relative to a model $\mathfrak{A}^n$ if the denotation of *John* in $\mathfrak{A}^n$ is an element of the set denoted by *walk* in $\mathfrak{A}^n$:

(1) *John walks*



In this sense, model theory provides for a truth-conditional (or denotation-conditional) characterization of the meaning of contingent sentences (as well as their parts). But it is obvious that this set up does not provide for a distinction of semantics (literal meaning of expressions) and pragmatics (use of the expressions by a speaker relative to a context).

1.1. Definition of a model structure $\mathfrak{A}$ and its models $\mathfrak{A}^n$

(1) illustrates the most basic type of model $\mathfrak{A}^n$ =df. (A,F), where A is the set of individuals (or entities), and F is a denotation function which assigns each constant of the language (e.g. *John*, *walks*, *unicorn*, etc.) an element of A^2 (i.e. the power set over A) whereby the semantic type ot the denotation must correspond to the category of the constant. This kind of model is also suitable for the definition of the usual quantifiers $\forall x$ (for all x) and $\exists x$ (there is at least one x), which are defined and interpreted relative to the model. The system is limited insofar, however, as it allows to model only one stationary situation. Some logicians, e.g. Quine, see no need to add a time structure to this stationary model because they are interested in modelling scientific truth, which is seen as constant.

If we want to model the history of science or if we want to account for the tense system of natural language, on the other hand, the stationary model $\mathfrak{A}^n$ does not suffice. In order to relativize situation descriptions to different moments of time and possible worlds, the above model $\mathfrak{A}^n$ may be expanded into the so-called model structure $\mathfrak{A}$ = df. $(A,I,J,\leq,F)$ (e.g. MONTAGUE, 1973). Thereby A and F are defined as in $\mathfrak{A}^n$, I is a second basic set, regarded as a set of possible worlds, J is a third basic set, regarded as a set of moments of time, and $\leq$ is the linear ordering on J (so that for any two moments of time j_1, j_2, we can say whether j_1 is earlier than j_2 or not).

Technically speaking, the model structure $\mathfrak{a}$ is simply a set of models which are arranged in terms of the index-structure constructed as the cartesian product of $I \times J$. Thus a sentence like $John\ walks^{\mathfrak{a},i,j,g}$ is interpreted not only relative to a model structure $\mathfrak{a}$ (and a variable assignment function g), but also relative to an index (i,j) in the model structure $\mathfrak{a}$. (i,j) renders the model $\mathfrak{a}^n$ in $\mathfrak{a}$ relative to which the sentence is to be interpreted.

A model structure $\mathfrak{a}$ of the indicated kind permits the definition not only of the quantifiers $\forall x$ and $\exists x$ relative to A, but also of the tense operators W ('it will be the case') and H ('it has been the case') relative to J, and the modal operators $\Box$ ('it is necessary') and $\Diamond$ ('it is possible') relative to $I \times J$. It allows furthermore the definition of so-called intensions. An intension is a function from the index-set into extensions:

(2) intension: $I \times J \longrightarrow$ extension

For example, the property of walking (i.e. a certain intension) is defined as that function which at each index (i,j) renders the extension of *walk*, i.e. the set of walkers in the model at that index. And a proposition (i.e. the intension denoted by a sentence) is defined as a function from $I \times J$ into the truth-values.

Model structures permit a refinement in the denotation-conditional description of natural language meanings (as compared to the extensional model $\mathfrak{a}^n$ defined above). For example, they allow a formal reconstruction of the Fregean distinction between sense (Sinn) and denotation (Bedeutung) in terms of the notions 'intension' and 'extension', respectively. This, in turn, permits the treatment of so-called intensional predicates (cf. MONTAGUE, 1973) or opaque contexts. As far as the distinction between semantics (literal meaning of expressions) and pragmatics (use of the expression by a speaker relative to a context) is concerned, however, intensional model structures $\mathfrak{a}$ fail in the same way as the extensional models $\mathfrak{a}^n$.

1.2. Reference and the ontological question

Model theory as described so far is very well suited to formally describe certain aspects of natural language meaning. But how should it be expanded to handle vagueness? In order to answer this question we must first clarify

(3a) what the model structure $\mathfrak{a}$ with its models $\mathfrak{a}^n$ is supposed to stand for, and
(3b) how reference, i.e. the relation between an expression and the object referred to, is supposed to come about.

These two questions are clearly related. Reference is usually defined as the relation between a language expression and the corresponding object of the model structure. If the model structure is interpreted as a representation of (actual or - in modal systems - possible) reality, then reference constitutes the whole relation between the language expression and the objects of the (model-theoretically simulated) real world. If semantics deals with the complete meaning connection between expressions and the world, then there is no room for a separate pragmatic analysis and reference is part of semantics.

Alternatively, let us consider the possibility of a system where reference is not the only and whole connection between the language expressions and the objects of the world. In such a system, the model-theoretic objects could be interpreted as *concepts*, standing for the real things (at least in certain instances) but not identical with the real things. If we make this assumption, then reference may be defined as a subsegment of a complex mapping from expressions to objects of reality. Candidates for such a submapping are (i) the relation between the expression and the concept and (ii) the relation between the concept and the real object.

In as much as we are dealing with concepts, it seems natural to assume that part of the real-token/real-object mapping is constituted by the information processing inside the speaker/hearer. It will become apparent that this particular choice of an ontology for the model structure (i.e. this specification of what the model structure is supposed to stand for) is of greatest importance for the way the original deduction system is to be expanded to handle phenomena arising with natural language, such as non-literal uses, context-dependency, propositional attitudes, etc.

Our two assumptions, namely (i) that the model structure represents something conceptual, and (ii) that the model structure is regarded as part of the speaker/hearer (formalized as a speaker simulation device or SID), not only harmoniously complement each other, but also render a number of natural implications which immediately lead to a much more specific notion of which submappings the real-token/real-object mapping is composed of. One consequence is a distinction between the SID-external reality and its representations inside the SID, whereby the latter is called the (SID-internal) *context-model*. The correspondence of the context-model with the outside reality is described by the submapping called *perception*. The context-model is also determined by a second input component, called the SID-internal memory.

A second consequence of our SID-based token representation is the distinction between the real token and the SID-internal token representation. The correspondence between the real token and the token representation is described by the submapping called *verbal processing*. Verbal processing is called articulation if the real token is a replica of the SID-internal token representation. Verbal processing is called recognition if the token-representation is a replica of the external token. The SID-internal token representation differs from the real token in that (i) it incorporates only the linguistically relevant properties of the token surface and in that (ii) it includes in addition a logical (model-theoretical) representation of the literal meaning of the token surface, which we call the *token-model*. We say that the surface of the token representation denotes its token-model(s). The token model will be defined in section 3.1 and is regarded as a set-theoretic icon of the literal meaning associated with the token surface.

Verbal processing is obviously the first segment of the real-token/real-object mapping, while perception is obviously the last segment of this mapping (assuming an SID-based ontology). Each of these two segments provides us with an SID-internal conceptual structure, one called the token-model, the other called the context-model. We complete the real-token/real-object mapping by defining a subsegment relating the token-model and the context-model. This subsegment is called *pragmatics* and defined in terms of matching the token-model and the context-model. Part of this matching relation is reference. We distinguish different types of pragmatic matching, such as what we call *literal use* (defined as an exact correspondence between the token- and the context-model, i.e. there is a proper embedding of the token-model in the context-model), *ironic use* (defined as a correspondence with striking contrast between the token- and the context-model), *metaphoric use* (defined as a correspondence of analogy between the token- and the context-model), etc.

Note that our SID-based reconstruction of the real-token/real-object mapping renders two notions of meaning (in accordance with HAUSSER, 1980b, 1981a, b, 1982). They are meaning^1, defined as the compositionally encoded literal meaning of the token surface, and meaning^2, defined as the speaker meaning of the utterance. The need to distinguish between these two types of meaning becomes obvious when we consider the literal and the ironic use of a sentence like 'That's real nice weather today'. We say that this expression has the same literal meaning (icon) in both use situations, but this icon is used to convey different speaker meanings. We relate meaning^1 and meaning^2 in terms of the following formula:

(4) use of $\left\{\begin{matrix} \textit{form} \\ \text{meaning}^1 \end{matrix}\right\}$ relative to a context = meaning^2

Thereby, the *form* is the SID-internal representation of the token surface, *meaning*^{1} is the correlated token-model, *context* is the SID-internal context-model, and *use* is defined as the matching of the token- and the context-model (cf. section 3.1 below for details), whereby properties of the token surface may also play a role in the pragmatic interpretation (cf., for an example, HAUSSER, 1982: 127).

1.3. A comparison of paradigm I and paradigm II systems

Let us call a system based on the traditional (realistic) model-theoretic ontology a paradigm I system and a system assuming our SID-based ontology a paradigm II system. Paradigm I systems assume that the model structure is a representation of reality and that reference is a direct relation between the real token and the real object. Paradigm II systems assume that there are two model structures, one for representing the literal meaning of the language token in question, the other for the representation of the SID-internal utterance context. As a consequence, paradigm II systems construct the real-token/real-object relation as a complex mapping, consisting of verbal processing, pragmatics (including reference), and perception. Consider (5a) and (5b), where the differences between paradigm I and paradigm II systems are represented schematically:

(5a) Paradigm I

real token

denotation
(= reference)

real object
(model-theoretic reality)

(5b) Paradigm II

VERBAL PROCESSING

real token ⊣ ⊢ *token representation*

denotation

token model

PRAGMATICS
(reference)

PERCEPTION

real object ⊣ ⊢ *context model*
(outside reality)

According to paradigm I, there is only one notion of truth, reference and denotation are the same, and there is no distinction between the objects denoted by language expressions and the objects of reality. According to paradigm II, on the other hand, there are altogether five notions of truth (as will be explained in section 2.1 below), reference is defined as part of the token-/context-model matching, and denotation is defined as the relation between the token surface representation and the token meaning, represented in terms of the token model.

1.4. Some problems of the paradigm I approach

The most pressing question raised by the paradigm I system is: Where does the speaker/hearer come in? Since the model structure is interpreted as a representation of reality, the speaker(s) must be part of the model structure. But what about cases where the speaker enters into the model-theoretic interpretation of language, such as the interpretation of indexicals. The standard proposal to extend paradigm I systems to a treatment of personal pronouns like *I*, *you*, *we*, adverbs of time and space like *here*, *now*, *this*, etc., studiously avoid any specifics on the "speaker/hearer-question".

This is exemplified by the so-called coordinates approach (Montague, Lewis), where the meaning of such pronouns as *I* and *you* is specified arbitrarily by additional model-theoretic parameters S (for speaker) and H (for hearer). Thus a sentence like *I am hungry* is interpreted relative to a model-structure $\mathfrak{a}$, a point of reference (i,j) (c.f. section 1.1 above), and furthermore relative to a speaker s ($s \varepsilon S$) and a hearer h ($h \varepsilon H$). (In the case of *I am hungry* only the value of s is truth-conditionally relevant, because only the pronoun *I* occurs in the sentence (no *you*!).)

On the one hand, this treatment of indexicals is clearly within paradigm I model-theory. But what is the theoretical nature of reference in this system? Intuitively, reference is sometimes equated with what we observe to be true (Carnap), but technically speaking the coordinates approach does nothing more than *assign* referents to context-dependent expressions. This assignment is by definition and thus arbitrary. Consequently, there is no natural way to treat contextual interrelations among indexicals within the coordinates approach. Such interrelations are constituted by the fact that, e.g. *I* means *you* in the ears of the hearer, while in the mind of the speaker *I* means *I*; and conversely, *you* means *I* in the ears of the hearer, but *you* means *you* in the mind of the speaker (for a more extensive discussion see HAUSSER, 1980b: 197ff.).

Another problem with the absence of the speaker/hearer in paradigm I model theory is the analysis of so-called non-literal uses, such as ironic, metaphoric, etc., uses. Since there is only one notion of meaning (if at all), defined as a direct relation between expressions and the model-theoretic reality, the only way to treat such non-literal uses is by postulating syntactico/semantic ambiguities. But analyzing the ironic use of, e.g., *That's really nice weather* logically as *The weather is not so nice* amounts to overextending *ad absurdum* the notion of syntactic ambiguity, i.e. an ambiguity caused by the syntactic structure of the surface expression, as in *Flying planes can be dangereous* (CHOMSKY, 1965) or *They don't know how good meat tastes* (CHOMSKY, 1966).

A third problem characteristic of paradigm I model theory is the treatment of propositional attitudes. For example, the sentence 'John believes that Cicero denounced Catiline' implies 'John believes that Tulli denounced Catiline' only if the sentence 'John believes that Cicero is Tulli' is true. This means that in order to adequately treat this inference the paradigm I model structure must describe not only the objectively given real and possible worlds, but also the subjective belief-worlds of all the speakers and hearers it contains. For an alternative solution within the paradigm II see HAUSSER (1982: 39ff. and section 7).

Last but not least consider the problem of treating vagueness in the paradigm I model theory. One proceeds by assuming the vagueness of natural language concepts, treated in terms of different degrees of truth (or absence of truth), and then constructs systems which assign to a complex sentence a fuzzy truth-value, computed from the fuzzy truth-values of the parts (and similarly in systems which use a third or undefined truth-value). This amounts to the same trivialization of reference as the treatment of indexicals (*I*, *you*, *this*, *now*) in terms of additional model-theoretic parameters. In either case, the emphasis is on the compositional aspect (i.e. on what happens if a word has a certain indexical interpretation or if a word has a certain vague extension), but the question of how an indexical or vague word obtains its particular value is treated as a matter of definition.

2. *On the nature of truth*

Let us now turn to the paradigm II model theory (cf. (5b) in section 1.3 above). As explained in sections 1.2 and 1.3, paradigm II systems assume two model structures, one for the set-theoretic characterization of the literal meaning of the token, the other for the set-theoretic characterization of the speaker context. Both of these model structures are assumed to be part of a formalized speaker/hearer, also called a speaker simulation device or SID. Outside the SID we may imagine a third model structure, representing the real world (and thus corresponding to the paradigm I model structure).

But we are not interested in a formal representation of the current notion of scientific truth or whatever else is considered to be really "real". Rather, we are interested in an operational simulation of natural language communication. Therefore, we would prefer the *construction* of an SID in order to observe its interaction with the real world (and not a model-theoretic representation of the real world). For this task, however, the efforts in Artificial Intelligence to design systems for artificial perception and perception analysis must advance. In the meantime let us consider the information processing inside the SID, specifically as it pertains to verbal communication. Thereby we take the process of verbal recognition and articulation as well as the construction of the context for granted and concentrate on the denotation conditional characterization of the literal meaning of the tokens and their pragmatic interpretation relative to a presumed context-model.

2.1. *Five notions of truth*

The paradigm II systems retain the formal methods of truth-conditional semantics, as originally developed within the paradigm I model theory. But what is the nature of truth in the paradigm II systems? The two models inside the SID, in their correlation to reality and to each other, render altogether five different kinds of truth. Three of those five kinds of truth are what we call *matching truth*, the remaining two kinds of truth are what we call *logical truth*. Let us consider these five notions of truth one by one.

The first kind of matching truth is *perception truth*, defined as the matching of non-verbal concepts in the context model with properties of the objects perceived (disregarding for the moment the proper storing and recall of memory, which is another factor in the build up of a "truthful" context model). The external objects of perception are called real or objects of reality, whereas their representation in form of (set-theoretically[1] defined)

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1 That we use set-theoretically based model theory for these representations, rather than, e.g., net-work semantics or procedural semantics, is at the present point mainly a matter of convenience and tradition. One may argue, however, that set-theory is the most general and most elementary form of

concepts in the context-model is something mental or conceptual. Accurate non-verbal perception is surely one prerequisite for arriving at truth in the philosophical sense (cf. section 2.3).

The second kind of matching truth (i.e. truth defined in terms of pattern matching) is what we call *verbal processing truth*, defined as the matching of the SID-internal token surface with the relevant properties of the external token. The external token is the real token, whereas the SID-internal token representation (of a real or potentially real token) is something mental or conceptual. Accurate verbal recognition as well as articulation is surely another important prerequisite for communicating truth, i.e. for a sentence said or heard to be true.

The third kind of matching truth is what we call *pragmatic* or *iconic truth*. Iconic truth is defined as the matching of the SID-internal token-model (i.e. the set-theoretic icon of the token) with the SID-internal context-model. We distinguish different characteristic types of iconic matching, such as literal use, ironic use, metaphoric use, etc., which in turn underly different types of iconic truth. The correct application of the pragmatic matching rules is surely a further prerequisite for communicating in a truthful way. This point is illustrated by those not infrequent situations where an ironic statement is interpreted as literal or vice versa.

Besides the three types of matching truth just described, we recognize two types of *logical truth*. One kind of logical truth is used in the construction of the token-model, the other kind of logical truth is used in the construction of the context-model. Of course, we may decide to use the same kind of logical truth in the construction of the token- and the context-model, just as we may decide to use the same type of matching truth in the analysis of perception and verbal processing. But at present we do not wish to preclude the possibility that the logic of the token-model is based, say, on a three-valued system based on partially defined functions, while the logic of the context-model is defined as a bivalent system or a supervaluation system.

Formally, there are many different ways to define logical truth. But the question is: what is the conceptual nature of logical truth, in contrast to matching truth? Let us consider a simple example, namely the definition of $\wedge$ (i.e. the logical operator which formalizes the meaning of English *and*) in the style of MONTAGUE (1973).

(6a) Syntax of $\wedge$: If $\phi, \psi \in ME_t$, then $[\phi \wedge \psi] \in ME_t$ (where ME_a is the set of meaningful expressions of type a and t is the type of formulas (sentences)).

(6b) Semantics of $\wedge$: If $\phi, \psi \in ME_t$, then $[\phi \wedge \psi]^{\mathfrak{a},i,j,g}$ is 1 if and only if $\phi^{\mathfrak{a},i,j,g}$ is 1 and $\psi^{\mathfrak{a},i,j,g}$ is 1.

The index $\mathfrak{a},i,j,g$ in (6b) consists of the model structure $\mathfrak{a}$ defined in 1.1 above, the possible world i, the moment of time j, and the variable assignment function g. The 1 mentioned in (6b), furthermore is an abstract model-theoretic object which is interpreted as "true", in contrast to the 0 (mentioned for example in the definition of logical negation), which is interpre-

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semantics, and that net-work and procedural semantic analyses may be translated into set-theoretic representations. Unfortunately, very little is known so far as to how these three types of systems compare; cf. ANDERSON (1976: 231ff.), where the net-work grammar ACT is translated into first order predicate calculus.

ted as "false"[2].

Now, what is logical truth? Since the power set A^2 over A (i.e. the set of entities in $\mathfrak{A}$) contains by definition $\emptyset$ (i.e. the empty set) and $\{\emptyset\}$ (i.e. the set containing the empty set), 0 and 1 are usually defined as $\emptyset$ and $\{\emptyset\}$, respectively. Thus A^2 provides the (extensional) denotation for expressions of all types and no additional objects (serving as truth values) need to be assumed in the definition of the model structure.

2.2. *Truth defined in terms of satisfaction*

The completely abstract nature of logical truth is somewhat veiled by the Tarskian definition of truth in terms of satisfaction. This widely accepted method is also used in the definition presently under discussion. The crucial clause (cf. MONTAGUE, 1973: 259) is repeated in (7):

(7) *Truth defined in terms of satisfaction*: If ϕ is a *formula* (that is, member of ME_t), then ϕ is *true* with respect to $\mathfrak{A},i,j$ if and only if $\phi^{\mathfrak{A},i,j,g}$ is 1 for every $\mathfrak{A}$-assignment g.[3]

According to clause (7) only *closed* sentences may be called true (false), whereas open sentences are called 1 (0) relative to an $\mathfrak{A}$-assignment g. It should be obvious, however, that this meta-language definition of 'true' in (7) is based completely on the abstract model-theoretic object 1 (or $\{\emptyset\}$) (plus the notion 'every $\mathfrak{A}$-assignment') and thus leads in no way beyond the abstract nature of logical truth defined as $\{\emptyset\}$.

In other words, the logical truth values, however defined, are essentially fix points in an abstract system of definitions. Note that this system of definitions is independent both of (i) reality and (ii) language (including the meta-language). In order to show this consider once more definition (6). (6b) characterizes the truth value of $[\phi\wedge\psi]$ relative to *any* model-theoretic situation, thus no specific reality is presumed. Indeed, the computation of $[\phi\wedge\psi]^{\mathfrak{A},i,j,g}$ from $\phi^{\mathfrak{A},i,j,g}$ and $\psi^{\mathfrak{A},i,j,g}$ does not depend on the assumption of any reality at all.

The second point may be a bit harder to grasp. It is a widely accepted truism that truth can be defined only relative to a meta-language. This truism seems to be supported also by definition (6b). However, an operator like '$\wedge$' becomes independent from the meta-language definition the moment it is implemented as a *computational operation*. As an analogy, consider the definition of '+' (addition) in arithmetic. The definition of '+' is also done in a meta-language, just as in the case of '$\wedge$'. But any simple pocket calculator "knows" the meaning of '+' without any recourse to a meta-language. Of course, a meta-language may be used to *build* the calculator, but once the definition of '+' is operationalized, we can forget the meta-language.

Thus, when we say that *logical truth* is (i) an abstract model-theoretic object which (ii) serves as a fix point of a system of abstract definitions, which (iii) is independent of reality, and which (iv) is independent of language, including the meta-language, then we presuppose that a paradigm II system has been concretely implemented in form of a speaker simulation device or

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2 See for example MONTAGUE (1974: 258): "Further, we identify the truth values falsehood and truth with the numbers 0 and 1, respectively".

3 Thus ϕ is true if it is satisfied by any sequence of variable values. For a more extensive discussion see for example BELL/MACHOVER (1977: 51ff.).

SID. Point (iii) and point (iv) may not hold of logical truth as used in paradigm I systems. But it should nevertheless be obvious that logical truth, as discussed in connection with (6) and (7), is conceptually something completely different from matching truth.

We will return in section 2.3 to the question whether paradigm I systems with their one notion of truth (i.e. logical truth) or paradigm II systems with their five notions of truth provide a more adequate characterization of truth in the philosophical sense. In conclusion of the present section let us return to the correlation between meaning and truth in the two paradigms. In the paradigm I systems, the model-theoretic objects are simultaneously the meanings of expressions and the objects of reality. The absurdity of this inherent assumption of the paradigm I approach has not gone unnoticed. But the remedy suggested again and again in analytic philosophy is even more absurd, namely the denial of the existence of meanings (as theoretical entities). In a paradigm II system, on the other hand, at least two notions of meaning are assumed. One is meaning^1, defined in terms of logical truth and regarded as the literal meaning of the token. The other is meaning^2, defined in terms of matching truth and regarded as the speaker meaning of an utterance.

2.3. Truth defined in terms of correspondence versus coherence

Adherents of paradigm I model theory may argue that a system with only one kind of truth (and no notion of meaning) is inherently better than a paradigm II system with its five kinds of truth (and two notions of meaning). Such a claim may draw strength from two quite opposite sources. One is a reasoning along Occam's razor: don't postulate entities for which there is no real need. But we have shown that our five kinds of truth perform functions which are intuitively clear and obviously necessary in an SID-based analysis. Besides, we could maintain that we are really using only two kinds of truth, namely matching truth and logical truth. However, while verbal processing truth and perception truth are in all likelihood based on the same matching mechanisms, pragmatic or iconic truth must probably be regarded as a separate type. Thus assuming that we use the same kind of logical truth for the construction of the token-model and the context-model, we end up with three main types of truth, namely (i) logical truth, (ii) iconic truth, and (iii) what we might call generalized perception truth.

Another argument for paradigm I model theory (and its one notion of truth) might appeal to the philosophical belief that there is only one truth. I do not wish to argue against this philosophical view. But the question is whether paradigm I systems with their one notion of truth, namely logical truth defined as, e.g., $\{\emptyset\}$, provide the proper formalization of what we might call philosophical truth.

In Western philosophy, truth is commonly defined in two basically different ways. One is the definition of truth in terms of *correspondence*, the other in terms of *coherence*. Thereby it is unclear and controversial how the two notions relate (see for example HACKING, 1975: 130f.). In order to maintain the view that there is only one truth, it is frequently suggested that the two definitions are simply different names for - or emphasize different aspects of - the same thing. In contrast, I would like to suggest that correspondence truth and coherence truth represent two essentially different kinds of truth, whereby correspondence truth correlates with our notion of matching truth, while coherence truth correlates with our notion of logical truth.

And indeed, it seems that paradigm I systems are really only suited for a treatment of coherence truth, that is, the treatment of logical deduction and logical consistency. Regarding the Aristotelian notion of truth in terms of correspondence, on the other hand, paradigm I systems are singularly unsuited to treat it, despite claims to the contrary (e.g. TARSKI, 1944). The reason is quite obvious: the paradigm I approach fails to provide anything that could correspond because the elementary meanings and the elementary objects of the (model-theoretic) situation are identified. Consider for example the so-called picture theory of WITTGENSTEIN (1922), where logical proper names stand for the things they name. Any such theory that identifies the denotation of elementary logical expressions with the objects of reality (i) trivializes the problem of reference and (ii) fails to provide a context-independent characterization of the literal meaning of expressions. And without a theory of expression meanings it is impossible to define a correspondence between expression meanings and the states of affair referred to. Note that the dictum 'A sentence is true if it corresponds to reality' may either be interpreted in the sense that *it* refers to the independently defined *meaning* of the sentence (in which case the dictum is in conflict with the paradigm I approach) or in the sense that *it* refers to the surface of the sentence (in which case the dictum is obviously false).

2.4. The paradigm I approach as a special case of the paradigm II approach

How do paradigm I systems and paradigm II systems relate? The paradigm II approach treats *coherence truth* in terms of the logical systems generating the token-model and the context-model, and *correspondence truth* in terms of the matching relation between reality and context, token and token representation, and token-model and context-model. Yet there is a quite natural way to relate paradigm I systems to paradigm II systems: if we assume that verbal processing in the paradigm II system is so accurate that the distinction between the real token and the token representation can be neglected, and that perception is so accurate that the distinction between the real world situation and its representation in the context model can be neglected, and that the use of language is so simple-minded that it consists only of the most literal use so that the distinction between the token model and the context model can be neglected, then we end up with a paradigm I system. In other words, the paradigm I systems are nothing but a special case of the paradigm II approach.

Another way to compare paradigm I and paradigm II systems is the following. Both, paradigm I and paradigm II systems relate the real token with the real object. But whereas in paradigm I systems the real-token/real-object relation is treated as a direct relation (with the result that the model structure is treated as a representation of reality of which the speaker/hearer(s) is (are) a part, paradigm II systems take this relation apart into several submappings by routing it through the speaker/hearer (with the result that the model structure is used to describe something conceptual which is part of the speaker/hearer).

The special case of a paradigm I system may be the proper choice when model theory is applied to systems of science, i.e. when reference (i.e. the relation between the elementary constants and the corresponding objects in the model-theoretic simulation of the real world) is presupposed to be accurate and logic serves only to check the consistency of the theory. But paradigm I systems are inappropriate when the goal is a model-theoretic analysis of communication. This is also true in the case of vagueness. Properly speaking, the treatment of vagueness in paradigm I systems is an absurdity: First formal logic and model theory are developed to escape the vagueness of natural language. Then the same system is "expanded" to handle vagueness, but without any change in the basic assumptions of the original program. How can there be vagueness in a formal system where reference is fixed by definition?

3. *Vagueness and presupposition failure*

On the paradigm I approach, the model structure is defined as a complete representation of actual and potential reality. The interpretation of a sentence relative to the model structure and an index consists in *checking* whether the sentence is true or not in the situation at the index. On the paradigm II approach, on the other hand, the model structure is partial in the sense that it characterizes only the semantic interrelations among logical constants (for discussion cf. HAUSSER, 1981b) without providing complete model-theoretic situations given prior to the interpretation of sentences. Rather, the semantic interpretation of a sentence in the paradigm II approach consists in *synthesizing* or constructing a model-theoretic situation (on the basis of the partial model structure) which makes the sentence true. This so-called token-model is then pragmatically interpreted in terms of its match with the context-model.

3.1. *The formal nature of the token-model*

But what exactly is the token-model? The most basic proposal (HAUSSER, 1981a) is to define the token model as a minimal model making the sentence in question true. By minimal we mean a model that is based (a) on finite domains and (b) assigns the smallest extensions to the logical constants still suitable for defining the matching relation between token-model and context-model in a simple and intuitively natural way.

However, if there are several minimal models making the sentence true, which one should be chosen to serve as the set-theoretic icon? Furthermore, the basic proposal as formulated above does not work (i) for contradictory sentences and (ii) for the semantic characterization of presuppositions. In case of a contradiction no model can be constructed which would make it true. And the semantic difference between, say, an existential assertion and an existential presupposition can be brought out only on the basis of models relative to which the sentence is false or undefined, but remains invisible if we limit ourselves to models which make the sentence true[4].

Let us therefore revise the basic proposal as follows: the literal meaning of a sentence is formally represented by the set of minimal models relative to which the sentence (i) is true, (ii) is false, and (iii) is undefined. Furthermore, the minimal models in this set must all be relevantly different. This set of models constructed for a given sentence *A* is called the *token model* or the *characteristic model set* of *A*.

Consider for example sentence (8),

(8) John walks and talks.

which translates into (8'):

(8') walk'(j') $\wedge$ talk'(j').

The characteristic model set of (8') consists of the following four minimal, relevantly different models:

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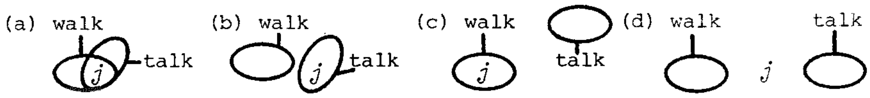
4 This problem was first pointed out in HAUSSER (1981b: Fn. 2). The following proposal to define the token-model as a set of characteristic models is a first informal solution of the issue raised in the above mentioned footnote.

(8")



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In a more formal way, this characteristic model set may be defined as follows:

(8*) the model set $\mathfrak{a}^{s}$ =df. (A, F^{s}), for s = a, b, c, or d.
A =df. $\{a_0\}$

(a) $F^{a}(j') = a_0$	(b) $F^{b}(j') = a_0$	(c) $F^{c}(j') = a_0$	(d) $F^{d}(j') = a_0$
$F^{a}(\text{walk}') = \{a_0\}$	$F^{b}(\text{walk}') = \emptyset$	$F^{c}(\text{walk}') = \{a_0\}$	$F^{d}(\text{walk}') = \emptyset$
$F^{a}(\text{talk}') = \{a_0\}$	$F^{b}(\text{talk}') = \{a_0\}$	$F^{c}(\text{talk}') = \emptyset$	$F^{d}(\text{talk}') = \emptyset$

Values: 1 0 0 0

For reasons of simplicity and graphical vividness we will in the following use the graphical method illustrated in (8") rather than the definition method illustrated in (8*).

As another example consider sentence (9),

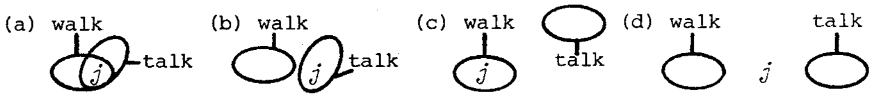
(9) John walks or talks.

which translates into (9'):

(9') walk'(j') v talk'(j')

The characteristic model set of (9') is like (8"), except that (9') is 1 relative to three minimal models and 0 relative to one, as indicated in (9"):

(9")



Values: 1 1 1 0

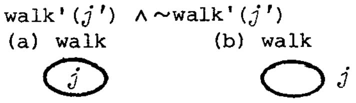
Next consider the characteristic model set of the contradiction (10),

(10) John walks and doesn't walk.

which translates into (10'):

(10') walk'(j') $\wedge\sim$walk'(j')

(10")



Values: 0 0

Thus a sentence is called a contradiction if its characteristic model set contains only "false models", i.e. models relative to which the sentence is 0. And a sentence is called a tautology if its characteristic model set contains only "true models".

It is an interesting logical problem to define an exact and explicit procedure which assigns to any well-formed formula its characteristic model set. The number of models in the characteristic set of a formula ϕ is a function of (i) the case distinctions in the definition of each operator occurring in ϕ, and (ii) the places of predicate constants occurring in ϕ. It is no accident, for example, that sentence (8) has 4 models in its characteristic set. They correspond to the 4 possible assignments to a formula of the form '$\phi \wedge \psi$'. And similarly in the case of example (9). For the moment we simply assume that there is exactly one characteristic model set for each well-formed surface expression (or rather its disambiguated *IL*-translation) and regard this model set as the *set-theoretic icon* of the expression, characterizing its compositionally encoded literal meaning (meaning[1]).

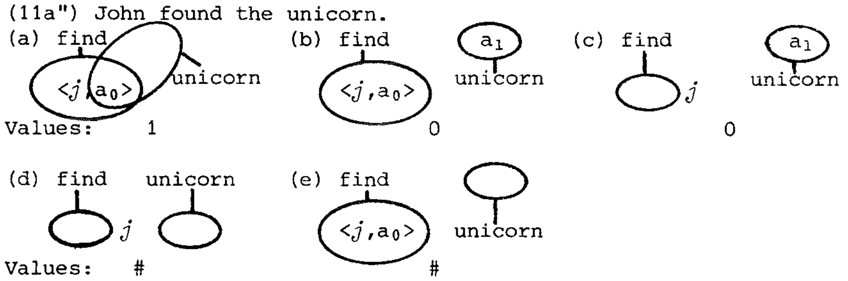
The interpretation of the token model as a characteristic model set extends naturally to the semantic characterization of presuppositions within the paradigm II approach. Semantic presuppositions are a truth-conditional property of certain natural language expressions, as witnessed by the comparison of (11) and (12):

(11a) John found the unicorn.
(11b) John didn't find the unicorn.
(11c) There is at least one unicorn.
(12a) John found a unicorn.
(12b) John didn't find a unicorn.
(12c) There is at least one unicorn.

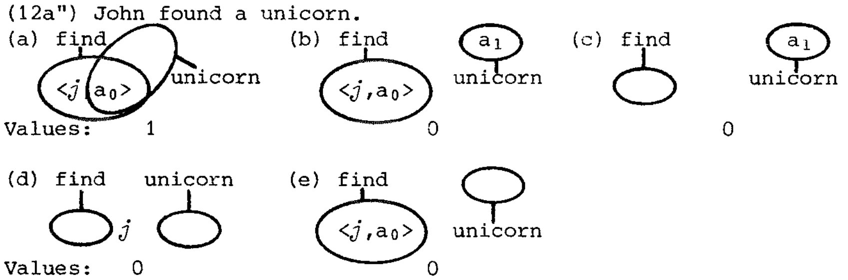
It is a simple fact of natural language that (11a) and (11b) both entail (11c), whereas (12b) - in contrast to (11a), (11b), and (12a) - does not entail (12c).

Standard presuppositional analysis treats these facts by assigning the value # (undefined) to (11a) and (11b) if (11c) is 0. In contrast, (12a) is 0 if (12c) is 0 and (12b) may either be 1 or 0 if (12c) is 0. These truth-conditional properties may be made explicit by the following characteristic model set (for the sake of simplicity, we leave the uniqueness condition associated with the singular of the existential *P*-inducer *the* (cf. HAUSSER, 1976) in (11) untreated):

(11a") John found the unicorn.



(12a") John found a unicorn.



The construction of the characteristic model set for a token sentence constitutes the *semantic interpretation* of the sentence. The *pragmatic interpretation* of the token, on the other hand, consists in finding that model of the characteristic set which matches best with the context. If the best matching model happens to be a model relative to which the token sentence is false or undefined, then there is no literal interpretation and a non-literal interpretation is attempted.

Of course, the notion of matching the token and the context model must be differentiated depending on whether we are dealing with an SID in the speaker state or in the hearer state. Consider for example a hearer receiving information by means of a token meant literally. For him the matching with the context consists in *incorporating* those models of the characteristic set into his context relative to which the token sentence is true.

3.2. *The role of the formal model in the two paradigms*

Note that the models indicated in (8"), (9"), (10"), (11"), and (12") are in no way a special feature of the paradigm II approach. Rather, exactly the same models are used in the paradigm I approach. Indeed, the models of the characteristic set of a token sentence and the value of the sentence relative to these models is motivated by presuming precisely that special situation which the paradigm I approach takes as paradigmatic, i.e. a situation where perception and verbal processing are abstracted from and the pragmatic parameter is frozen to literal use.

Thus it is only the intuitive interpretation of these formal structures that differs in the two paradigms. However, in a paradigm I system, as a consequence of its peculiar ontology, a formal model will be incomparably more complicated than those illustrated in (8") to (12") because the model must provide the extension of *all* constants of the language at a given index. On the paradigm II approach, on the other hand, the models in the characteristic set of a token assign extensions only to those constants which actually occur in the token sentence under semantic interpretation.

Furthermore, since on the paradigm II approach the models are interpreted as set-theoretic icons it is sufficient to construct *minimal* models, i.e. models assigning minimal extensions to the constants occurring in the token sentence under interpretation. The paradigm I approach, on the other hand, interprets the models as representations of reality. Therefore, the cardinality of the extension sets is not determined by the goal to generate all relevantly different models for a given token, but rather by the *situation* the model is supposed to simulate. Consequently, a paradigm I approach is either a vastly simplified reproduction of reality, and therefore unrealistic. Or the paradigm I approach attempts to be a realistic reproduction of reality, in which case its implementation would blow the mind of the largest computers presently available, since the addition of even one entity to the domain exponentially increases the amount of memory space needed.

3.3. *On the nature of presupposition failure versus vagueness*

Let us turn now to the nature of presupposition failure and vagueness in the two paradigms. It is characteristic of a theory which is a special case of a more general theory that certain distinctions which are well-motivated in the general theory collapse in the context of the special theory. In the case of paradigm I systems, which are a special case of paradigm II systems (as explained in section 2.4 above), this phenomenon may be observed in a number of linguistically highly relevant instances. The most general notions at issue are those of semantic versus pragmatic interpretation, which are completely distinct processes in a paradigm II system, but indistinguishable in a paradigm I system. A more special case in point are the notions of vagueness and presupposition failure.

In a paradigm I system vagueness and presupposition failure are essentially indistinguishable: Both arise when a sentence cannot be evaluated as either really true or really false. Therefore, the assignment of no truth value or a third truth value is assumed in either case (per definition of the model structure) and the whole interest is directed towards the question of what deductions are valid from premises with an undefined or third value, or how component sentences with an undefined or third value figure in the value of a complex sentence. The assumption that vagueness and presupposition failure are logically the same is explicitly made in BLAU (1977). Implicitly this assumption is made in KAMP (1975, 1981), PINKAL (1981), and others who use supervaluations, i.e. a system developed specifically for the treatment of presupposition failure (cf. VAN FRAASSEN, 1968), for the handling of vagueness[5].

In a paradigm II system, on the other hand, presuppositions are a semantic phenomenon, while vagueness arises as a pragmatic phenomenon. The semantic nature of presuppositions (and presupposition failure) is captured in a paradigm II systems in terms of the characteristic model sets (cf. (11) in section 3.1 above) of token sentences with *P*-inducers (i.e. presupposition-inducing words like *the*, *every*, *regret*, *stop*, etc., cf. HAUSSER, 1976). In contrast to presuppositions, which are a denotation-conditional property of expressions (as obvious from the comparison of (11) and (12) above), vagueness comes about as a property of *utterances*. For concrete paradigm II analyses of examples involving vagueness see section 4 below.

3.4. Three kinds of vagueness

What kinds of vagueness are there? There is verbal processing vagueness, i.e. cases where the SID can't recognize a token because of bad articulation or background noise or where the SID can't articulate properly. There is per-

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5 From a logician's point of view, the advantage of supervaluations is that reference may be defined bivalently. Thus, a predicate is always either true or false relative to a model-theoretic situation on its so-called "classical evaluation". The price, however, is the assumption that a predicate is to be evaluated *several times* relative to the same index. If the predicate is true in some such valuations and false in others, then the supervaluation (i.e. the valuation of the classical valuations) is undefined. Once we get to the level of supervaluations, van Fraassen's system works like the three-valued system of Kleene (cf. RESCHER, 1969), except that a sentence of tautological form, e.g. $A \vee \sim A$, is undefined for Kleene if A is undefined, whereas for van Fraassen the so-called classical tautologies are valid no matter whether their constituents are defined or not (and accordingly for contradiction).
The question of whether the classical tautologies should be always true or only if their constituents are defined seems particularly important within the paradigm I approach with its single notion of truth. In a paradigm II system, on the other hand, the truth of a statement depends on the accuracy of verbal processing, the accuracy of perception and memory, and the proper pragmatic interpretation (the intended *use* of the expression relative to the context). It is a fact of nature that even a logical contradiction may be used to make a true statement. Consider the sentence 'It rains and it doesn't rain', uttered in the dry desert where rain drops reach the hand but evaporate before they reach the ground. Statements like this are common and nobody would accuse the speaker of saying something nonsensical, or false.
Now, if a logically contradictory sentence may be pragmatically true, nothing much seems to be lost (as far as the characterization of truth - in the holistic sense - in a paradigm II system is concerned) if logical tautologies and contradictions are undefined if their constituents are undefined. Thus, a sentence is called a tautology if it is 1 relative to all those models in its characteristic set which fulfil the presuppositions of its constituent sentences. In other words, a sentence is a tautology, if its characteristic model set contains no models relative to which it is 0.

ception vagueness, i.e. cases where the SID can't recognize something clearly because of bad lighting or a hangover after a linguistic party. And there is what we might call iconic vagueness. Iconic vagueness is so-called not because the icon is vague, but refers to an uncertainty regarding the intended matching relation between the token and the context model.

We have thus arrived at an intuitive concept of vagueness which is quite different form the widespread view that "natural language concepts have vague boundaries and fuzzy edges". We conclude furthermore that vagueness must be associated with matching truth, while presuppositions are properly treated in terms of logical truth. It follows that the attempts to treat vagueness in terms of extensions of traditional logic not only vastly complicate the logic, either by assuming a large number of truth values (fuzzy logic) or by assuming a large number of evaluations of a predicate at a point of reference (supervaluations), but also completely miss the intuitive essence of linguistic vagueness. This is not to deny, however, a certain intrinsic value of these systems as sophisticated logical mechanisms.

4. *Examples of vagueness and their paradigm II treatment*

Let us turn now to the treatment of some concrete examples exhibiting vagueness. How should the slowly closing door be treated in a paradigm II system? The sentence 'The door is open' has a clearly defined literal meaning, formally represented by its token model synthesized on the basis of its standard *IL*-translation and defined as a characteristic model set. The characteristic set contains a model *T*, exhibiting a situation where the door is open, relative to which the sentence is 1, and a model *F*, exhibiting a situation where the door is closed, relative to which the sentence is 0. Observing the slowly closing door, at first model *T* will be the best match, then model *F*.

But what about the moment when model *T* and model *F* match equally well (or badly)? This is the situation problematized by treatises on vagueness within the paradigm I approach. Within the paradigm II approach, on the other hand, it is not a problem concerning the literal meaning of the sentence 'The door is open' and therefore it is not a logical problem. The question for us is rather how the sentence is *used* relative to the indicated situation. And there it seems that a normal speaker will simply use another sentence, like 'The door is closing', or wait a few moments and then say 'The door is closed'.

4.1. *The iconic content of red*

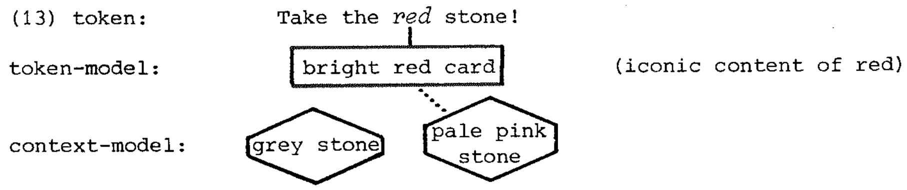
Next consider the sentence 'Take the red stone!', interpreted in the following two different situations. In one situation the hearer is confronted with a grey stone and a pale pink stone. Obeying the utterance 'Take the red stone!', he will pick the pale pink stone. In the other situation the hearer is confronted with a bright red stone and a pale pink stone. In this case he will not pick the pale pink stone but the bright red stone. Within paradigm I model theory it follows perfectly straightforwardly that the word *red* is vague: sometimes *red* is true of the pale pink stone and sometimes it is false of this same stone.

It has been suggested (NUNBERG, 1978) that predicates referring to different objects in different contexts can be handled in terms of "context-dependent functions". Thus the word *chicken* has the set of live chicken as its extension in one context, but the set of chicken meat batches in another context. This proposal remains firmly within the paradigm I approach in that it incorporates "context-dependency" (of expressions which are clearly not indexicals) into the semantics and treats the relation between, e.g. *chicken*, and its real referents as a direct semantic relation. In a paradigm II theory, on the other hand, the word *chicken* denotes one and the same icon in the two interpretation contexts and the different real world referents are accounted for in terms of different uses.

Similarly in the case of the pale pink and the red stones. The sentence 'Take the red stone!' is not ambiguous (for denotational conditional treatment of imperatives see HAUSSER, 1980a, 1983) and neither is the word *red*. But how should the iconic content of red be described? There are two aspects to the description of the literal meaning of an elementary constant like *red*. One is the set-theoretic interrelation with other logical constants of the same category (or semantic type). Thus the partial model structure of a paradigm II system will define that the extensions of *red*, *blue*, and *green* have disjunct sets as their extensions which are all subsets of the extension of the constant *color*. The other aspect concerns the specific difference between *red* and *blue* as opposed to the difference between *red* and *green*. Within the paradigm II approach, these distinctions may be treated naturally in terms of specific types of SID-perceptions. Thus the iconic content of *red* may be defined in terms of matching a certain wave-length of the electro-magnetic spectrum.

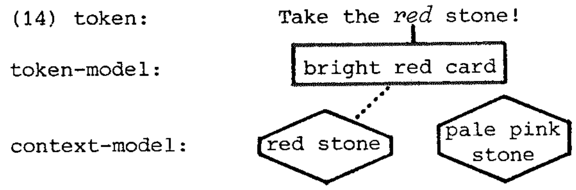
For the sake of simplicity let us assume that the iconic content of *red* is represented in form of a little card of bright red color (regarded as the SID-internal prototype of *red*). Then the interpretation of the sentence 'Take the red stone!' relative to the two situations described above may be indicated as in (13) and (14):

(13)



If we change to a context where the grey stone is replaced by a dark red stone, the pale pink stone ceases to be the one that matches the bright red card best. Thus we have a situation as indicated in (14):

(14)



So what happens to be the "red stone" in (13) turns out to be the "non-red stone" in (14). The point is that it is not the meaning of *red* that is vague or which changes, but rather it is the context which changes and thus the instances of best match.

4.2. The Sorites paradox

Next let us consider a classic paradox, namely the so-called Sorites paradox or paradox of the heap. This paradox brings out the essence of the paradigm I approach to vagueness. Thus it is not surprising that it received considerable attention by contemporary paradigm I logicians interested in vagueness. The paradox is described as follows. One grain of sand does not form a heap. If we add one grain, we still don't have a heap. If n grains don't form a heap, then adding an $n+1^{st}$ grain will not result in a heap. Yet at some point, when enough grains are added, we arrive at something that is undeniably a heap.

The recent proposals to resolve this paradox all accept it as a semantic paradox and thus stay within the traditional framework of semantics. But the price paid for these different kinds of so-called semantics of vagueness is considerable. KAMP (1981) arrives at a notion of semantic inference which is so far removed from the traditional notion that Kamp himself doubts as to whether his system may still be called a logic. KINDT (1983), on the other hand, proposes to incorporate the heavy machinery of mathematical topology into formal semantics, whereas PINKAL's (1982) approach of "precisification" constitutes a sophisticated development of the method of supervaluations.

These proposales have in common that they accept the premises which lead to the paradox. But when we look at another antique paradox, that of Achilles and the turtle, which today is regarded as solved, we see that one acceptable resolution of a paradox is to revise its premises in an intuitively convincing way. Indeed, this may be the only way to solve a genuine paradox. The moment we accept that a heap is to be defined in terms of a certain number of grains (e.g. 1 grain: no heap, 100 000 grains, properly arranged: heap) we are trapped. Because now comes the inevitable question: how many grains exactly make the difference between a heap and a non-heap?

4.3. *The icon of a heap*

So let us look at the problem in a different way. As illustrated by our discussion of the slowly closing door and the pale pink versus dark red stones, the crucial question within our paradigm II approach with regard to the paradox of the heap is: what is the icon of the word *heap*? And then: how is this icon used? Regarding the proper definition of the icon 'heap' we submit that it should not be defined in terms of a certain number of grains, not even upper or lower limits of this number. Rather, the icon of a heap is a prototype involving (i) a certain form (cone-like), (ii) a certain subsistence (loosely packed smaller parts), and (iii) certain proportions (the size of the smaller parts in relation to the size of the heap and the size of the heap in relation to the rest of the context).

Consider for example two people, called *A* and *B*, flying in 10 000 feet altitude, and *A* says to *B*: "That heap wasn't there yesterday" pointing to what looks like a tiny speck on the ground. In such a case, *A* would violate the proper use of the icon 'heap', even if it should turn out later that the speck on the ground was indeed a proper heap of sand. The speaker *A* may be *construed* to be right in a narrow, pseudo-scientific or pseudo-semantic sense, but that does not mean that *A* communicated in a natural and reasonable way.

Of course, if *A* were to say to *B*: "Do you see that tiny speck down there? That must be a heap of sand. I don't think it was there yesterday" the situation would be different (from a communication point of view). In the second case, the speaker *A* introduces a context-change. *A* leads *B* from point of view I (at 10 000 feet altitude) to point of view II (at the ground level close by). In the second (imagined) context the speck in question my well be a proper heap. It is of no consequence that *B* cannot *verify* *A*'s conjecture. All that is required is that *B* is a cooperative partner in this communication in the sense that *B* is willing to *provide* a context which accomodates the icon 'heap' (on the literal interpretation intended by *A*).

5. *Conclusion*

The difference between the paradigm I and the paradigm II approach to the Sorites paradox may now be summarized as follows. The paradigm I approach presumes a model-theoretic reality which provides various samples of heaps and non-heaps, starting from one lone grain and going up to a 100 000 grains, say. The supposed problem is to find a semantic definition of the logical constant *heap*, such that *heap*(x) is evaluated 0 (false) if x denotes only one grain, *heap*(x) is evaluated 1 (true) if x denotes the 100 000 grains, and which furthermore assigns the right truth-values in the critical transition from non-heap to heap. However, no semantic theory can fulfil this last desideratum, because the transition from non-heap to heap is intuitively unclear in a non-trivial sense. This intuitive problem with the traditional approach to vagueness is unsolvable because it derives from asking the wrong questions on the basis of the oversimplified and as such mistaken assumptions of the paradigm I approach.

On the paradigm II approach, on the other hand, there is no attempt to characterize the transition from non-heap to heap in the semantics. Rather, the icon 'heap' is semantically defined in a fixed way as a prototype, just as the icon 'red' was defined in terms of a little red card. The question of whether something is properly referred to as a heap or not is left to the pragmatic process of matching the icon with the context. What counts as a proper heap in one context, may be a definite non-heap in another context (just by changing the relative proportions of the objects relative to each other and relative to the context frame). This is similar to our example of a pale pink stone, which turned out to be the red species in (13) and the non-red species in (14). A further possibility, never even discussed in the paradigm I approach, is the metaphoric use of the icon 'heap', such as when an old car is referred to as a "heap of scrap". In this case, the icon invokes an imagined future state of desintegration which is felt to be so immediately pending as to justify this manner of speaking.

On the whole, we have argued that the paradigm II approach does more justice to the actual functioning of natural language than the paradigm I approach. The reason is that the paradigm II approach provides the distinction between the literal meaning of expressions (meaning^1) and the speaker meaning of utterances (meaning^2). This distinction collapses in the paradigm I approach, due to its being a special case of the paradigm II aproach (as was shown in section 2.4). According to the paradigm II approach, it is not the concepts of natural language (i.e. the meaning^1) which are vague. Rather, vagueness originates with the *use* of these concepts and is thus a meaning^2-phenomenon. The distinction between meaning^1 and meaning^2 in the paradigm II approach not only eliminates vagueness as a semantic problem, but also explains the flexibility and descriptive power of natural language: due to the fixed meaning^1 of language expressions (the so-called icon) we can describe phenomena and situations which were never known or described previously.

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