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PRESUPPOSITIONS IN MONTAGUE GRAMMAR

This paper deals with the question of how to construct a formal grammar of natural language which can compute all relevant presuppositions of any of its sentences. In other words, how can we find all relevant presuppositions of any given sentence on the basis of formal criteria and independent of our intuition? It is suggested that the presuppositions of simple sentences are *induced* by certain lexical items. Furthermore, in complex sentences specifically defined contexts may *cancel* presuppositions. In order to demonstrate the complementary principles of presupposition induction and presupposition canceling, certain English quantifiers (some of which are shown to be existential P-inducers) and sentential connectives (which are known to have canceling properties) are investigated. Regarding the analyzed linguistic phenomena a uniform semantic approach (in terms of partial functions and a propositional logic of the strong type) is advocated. Finally, the empirical results are implemented into the formal fragment of English described in Montague's PTQ.

1. Presuppositions of Simple Sentences

1.1. *Justification of a semantic treatment of presuppositions*

The linguistic literature on presuppositions in recent years distinguishes *semantic* presuppositions and *pragmatic* presuppositions (e.g. Keenan (1971), Stalnaker (1970), Karttunen (1973a)). The concept of a semantic presupposition goes back to Strawson (1950a) and may be explicitly characterized as follows (after Van Fraassen (1968)):

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[1] Some of the basic ideas incorporated in this paper were first presented in Hausser (1973). Asa Kasher has informed me that regarding the use of restricted quantification as well as the P-inducing properties of certain quantifiers he arrived at similar results. A brief outline can be found in Asa Kasher (1973). The present paper evolved out of earlier research I did for my dissertation. I am grateful to my teachers Stanley Peters, Lauri Karttunen, and Jack Murphy, who contributed greatly to the development of the ideas presented in this paper. Furthermore, thanks are due to Kiyong Lee, Marga Reis, Theo Vennemann, and Dietmar Zaefferer, who commented on an earlier version. Needless to say, the remaining mistakes are my own.

1) Sentence A presupposes sentence B if and only if A is neither true nor false unless B is true.

This (according to Van Fraassen) is equivalent to

2) Sentence A presupposes sentence B if and only if
   a) if A is true, then B is true;
   b) if (not-A) is true, then B is true.

While semantic presuppositions are defined as a relation between sentences, *pragmatic* presuppositions—as discussed by Keenan (1971) and Stalnaker (1970)—can be viewed as conditions on the *sincerity of an utterance*. Thus, if sentence A pragmatically presupposes B, then the speaker can utter A sincerely only if he takes the truth of B for granted.

Stalnaker points out that "there is no conflict between semantic and pragmatic presuppositions: they are explications of related but different ideas. In general, any semantic presupposition of a proposition expressed in a given context will be a pragmatic presupposition of the people in that context, but the converse clearly does not hold" (Stalnaker 1970). However, if Stalnaker is right and all semantic presuppositions are pragmatic presuppositions, why do we need semantic presuppositions at all? Couldn't we get by with just one notion of presupposition, namely the general case of pragmatic presuppositions? In order to answer this question we have to decide what truthvalue should be assigned to a sentence in case of a failing presupposition. Consider for example sentence (3):

3) The present king of France is bald

If we say that (3) is false—given that the presupposition fails and no king exists—, then shouldn't the negation of (3), i.e. (4), be *true*?

4) The present king of France is not bald

Yet (4) would hardly be called true if no king of France exists. In other words, even if all semantic presuppositions are also pragmatic presuppositions (which to me is an assumption of doubtful descriptive value), we do not get around the problem of assigning intuitively satisfying truthvalues to sentences like (3) in the case of a failing presupposition. In this dilemma we may choose between the following alternatives:

If a presupposition of sentence A fails, then

i) A is assigned no truthvalue,
ii) A is assigned an arbitrary bivalent truthvalue,
iii) A is assigned a bivalent truthvalue which is motivated within the framework of the particular analysis.

The first alternative represents Strawson's proposal and is explicated in the definitions (1) and (2). The second alternative is surely not very appealing as it stands.

It can be made part of an intuitively satisfying semantics, however, if "supervaluations" are employed (Van Fraassen 1968, 1969). Yet supervaluations are simply a more complex way of capturing Strawson's intuition—with the advantage of maintaining certain assumptions of classical logic. In other words, if we *extend* the second alternative in the sense of Van Fraassen, then it reduces to being a special case of the first alternative.

An example of the third alternative, finally, is Russell's analysis of the definite article (Russell 1905). Russell held that sentence (3)

3) The present king of France is bald

is *false* with respect to the actual world of (1905) or today. He motivated this bivalent truthvalue under the indicated circumstances by ascribing to (3) a logical representation like (5)

5) $Vx[\Lambda y[\text{king of F.}(y) \longleftrightarrow y = x] \wedge \text{bald}(x)]$

and by claiming that the negation of (3) is *ambiguous*. On the *narrow scope* negation reading sentence (4) is *false*—just like (3).

4) The present king of France is not bald

This reading is represented as (6):

6) $Vx[\Lambda y \text{king of F.}(y) \longleftrightarrow y = x] \wedge \sim\text{bald}(x)]$

(6) entails—just like (5)—that $Vx[\Lambda y[\text{king of F.}(y)]]$. On the *wide scope* negation reading, however, (4) is *true* (under the assumption that no king exists). This reading is represented as (7):

7) $\sim Vx[\Lambda y[\text{king of F.}(y) \longleftrightarrow y = x] \wedge \text{bald}(x)]$

In (7) the existential quantifier is within the scope of negation and thus (7) does not entail $Vx[\Lambda y[\text{king of F.}(y) \longleftrightarrow y = x]]$.

If no king exists, both (3) and (4) are *false*—assuming the narrow scope negation reading of (4). And still, if (3) is false, then (4) is *true*— assuming the wide scope negation reading of (4). In short, Russell dissolves the dilemma by postulating a structural ambiguity. The continuing appeal of Russell's analysis is due to the fact that it permits retaining the classic two-valued logic. Linguistically, however, Russell's analysis is untenable for the following two reasons:

i) sentence (4) is intuitively unambiguous,
ii) the analysis cannot be extended to other instances of presuppositions.

The first point will be discussed in more detail in section 1.4 below. In order to see the second point consider sentence (8):

8) John regrets that Mary left.

(8) intuitively presupposes (9):

9) Mary left.

Extrapolating Russell's approach to example (8) let us assume that (8) is false if (9) is false. Under this assumption we should expect that the *wide scope* negation of (8), i.e. (10), is *true*.

10) John doesn't regret that Mary left

The point is that intuitively (10) is certainly *not true* if (9) is false. Therefore a Russell-type analysis cannot be extended to examples like (8). Strawson's approach, on the other hand, handles the sentences (3), (4), (8), and (10) all in the same way: these examples are without truthvalue if the respective presuppositions fail.

The above considerations lead to the following conclusions: Even if semantic presuppositions are at the same time pragmatic presuppositions, we cannot avoid the question of what truthvalue should be assigned to a sentence in case of a failing presupposition. Since there doesn't seem to be a good way to "define away" the problem, and since an arbitrary assignment of truthvalues cannot be justified, the only acceptable alternative is a semantic treatment of presuppositions in Strawson's sense. This conclusion is based on the assumption that a model-semantic treatment should be part of the analysis of meaning in natural language. This is not to deny, however, that additional concepts like sincerity of the speaker may play an important role in a complete analysis of meaning.

1.2 A linguistic hypothesis regarding semantic presuppositions

One consequence of Strawson's definition of a semantic presupposition (c.f. (1) and (2) above) is that all *tautologies* are presupposed by every sentence. Therefore, any given sentence will have infinitely many semantic presuppositions. However, the truthvalue of tautologies does not vary with respect to different interpretations (i.e. tautological presuppositions are always fulfilled).[2] Therefore, in order to determine whether some sentence A is bivalent with respect to some point of reference *i*, it would be sufficient to check the truthvalue (with respect

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[2] It is actually a complicated question whether tautologies are always fulfilled in a presuppositional logic. The reason is that one might want to hold that something like (Av∼A) is valid only if A is *bivalent*. Note that under this assumption tautologies would not be presupposed by every sentence. The general point is, however, that tautologies do not affect the truthvalue of a contingent sentence and can therefore be disregarded. The problem of finding a basic set of presuppositions for any given sentence remains, however, no matter how we resolve the question of tautologies. It is my hypothesis that the set of P-induced presuppositions of any simple sentence A is in fact a basic set of A.

to *i*) of all presuppositions *other than tautologies*. Indeed, in order to determine the truthvalue of a sentence A, it is sufficient to only check the truthvalue of the members of a minimal set $\{B_1, B_2, \ldots, B_n\}$ of presuppositions of A such that all other presuppositions of A are entailed by $B_1 \wedge B_2 \wedge \ldots \wedge B_n$. Let us call such a set a *basic set* of A. The question now is: how can we determine the basic set of a given sentence without depending on our semantic intuition?

In order to answer this crucial question we have to know what causes certain sentences of natural language to have the particular presuppositions they have. There is evidence that the *relevant* (e.g. non-tautological) presuppositions of sentences of natural language *depend on the surface form* of the sentence: on its verbs, the types of noun-phrases, the presence or absence of modals, intensional predicates, etc. But since presuppositions are defined as a semantic relation between *sentences,* we cannot say, for example, that a particular verb "presupposes" something. Instead, let us say that such a presupposition guaranteeing component of a sentence *induces* a presupposition. We will call a presupposition guaranteeing component a *P-inducer*.

The basic linguistic hypothesis underlying the present approach to implementing presuppositions into a formal grammar of English may now be formulated as follows: I assume that the occurrence of relevant semantic presuppositions in natural language is systematically predictable. More specifically, I assume that all relevant presuppositions are induced by certain lexical items, called P-inducers. Thus in order to implement presuppositions formally we have to find out which lexical items are P-inducers, and for each P-inducer we have to state precisely which presupposition(s) it induces. Examples of P-inducers are the verbs **regret, stop,** and **know,** and the quantifiers **every, some,** and **the.**

Once we have established a set of P-inducers, we can study under which syntactico-semantic conditions presuppositions are canceled. I claim that the canceling of presuppositions depends on two very specific types of context. The first type consists of contexts which I would like to call 'syntactic contexts'. For example, opaque contexts created by the presence of an intensional predicate or complex sentences build up by the use of the English connectives **and, or,** or **if . . . then** represent syntactic contexts which may lead to the canceling of P-induced presuppositions. The other type of context are different *uses* of a sentence. Thus the presuppositions of a sentence may be canceled if the sentence is used ironically or as a question. The point is that the indicated types of canceling contexts are very specific and are open to a model-theoretic description.

Our approach leads naturally to a rather narrow concept of presupposition. In particular, it excludes cases where "presuppositions" arise solely out of what might be called the *situational* context. By 'situational context' I mean the outside circumstances of a given speech act which *do not affect the semantic interpretation* of a given sentence. Therefore the concept of presupposition advocated here stands in contrast to the notion of pragmatic presuppositions as recently proposed by Stalnaker and Karttunen. Stalnaker, for example, argues: "If presupposition

is defined independently of truth conditions, then it is possible for the constraints on presuppositions to vary from context to context, or with changes in stress or shifts in word order, without those changes requiring variation in the semantic interpretation of what is said. This should make possible a simpler semantic theory; at the very least, it should allow for more flexibility in the construction of semantic theories."

But what truthvalue would Stalnaker assign to sentences like (3), (4), (8), and (10), if the respective presuppositions fail? Furthermore, are there really presuppositions that arise only in some contexts? There should be strong evidence for this kind of phenomenon in order to justify the pragmatic approach. Stalnaker offers one single example (quoted from Langendoen):

11) My cousin isn't a boy anymore

According to Stalnaker, "if one said 'my cousin isn't a boy anymore' he would be asserting that his cousin had grown up, presupposing that he is male. But one might, in a less common context, use the same sentence to assert that one's cousin had changed sexes, presupposing that she is young." This seems to me a highly dubious concept of presupposition. (11) simply presupposes (12):

12) My cousin used to be a boy.

(12) covers both situational contexts described by Stalnaker. It is exactly those circumstantial factors induced by various situations which I want to exclude from my treatment of presuppositions. To me, P-induced presuppositions as well as the systematic canceling instances are constant, characteristic, inalienable properties of certain lexical items and (underlying) logical operators.[3] Since the

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[3] What then is the P-inducer in (11)? The non-negated version of (11) is presumably

i) My cousin is still a boy.

Should we treat the pair of polarity items **still/anymore** as a P-inducer? We could say that any sentence of the form (not-A anymore) or (A still) is truthvalueless unless HA is true, where H is the tense operator for the past tense. I am not sure, however, whether such a solution would be desirable in the long run. The finding of P-inducers is thus by no means a trivial linguistic undertaking. For example, is **too** a P-inducer? Consider example (ii).

ii) Nixon lied, too.

We could analyze **too** alternatively as a conjunction marker which marks both, underlying and overt conjunctions. Consider as an example of the latter (iii):

iii) Haldeman lied and Nixon lied, too.

Since **too** does not *always* induce the *implicit* assumption that someone else lied, I would derive (ii) from

ii a) Someone lied and Nixon lied, too.

rather than treating **too** as a P-inducer. A similar problem arises in connection with **only.**

instances of presuppositions (as exemplified by the examples (3), (4), (8), and (10)) are—according to my hypothesis—invariant with respect to situational contexts, they should *not* be subsumed among other (certainly interesting) phenomena relating to situational contexts, but warrant a separate analysis. This analysis should be in semantic terms, because the requirement of an intuitively motivated and justified truthvalue assignment (that is lack of truthvalue in case of presupposition failure) is at the heart of a model-semantic approach to the analysis of natural language.

1.3. Existential P-inducers and restricted Quantification

Let us turn now to the linguistic problem of *finding* P-inducers of a natural language. It would, of course, go beyond the scope of this paper to present *all* P-inducers of English. Rather, our investigation will be limited to certain *quantifiers* of English. By showing which quantifiers are existential P-inducers and which are not, I hope to demonstrate that the first part of my linguistic hypothesis (i.e. that semantic presuppositions are induced by certain lexical items) is linguistically plausible.

I will proceed by testing the quantifiers **every, any, some, the, a(n),** and the natural numbers with respect to the frames

John kissed [. . .] girl at the party
John didn't kiss [. . .] girl at the party

The test will consist in checking various existential entailments and presuppositions that arise with different quantifiers. After finding out which of the quantifiers are existential P-inducers, we will turn to the question of how existential P-inducers could in general be represented formally in the formulas of an auxiliary logical language. The logical language we are aiming at is the intensional logic of PTQ. However, in order to allow the reader to concentrate fully on the empirical as well as theoretical issues to be discussed, we will at first avoid a rigorous formal treatment and make due with the usual kind of first order predicate calculus. In the final sections 3.1 and 3.2, however, we will present our results in a rigorous formal way in form of an extension of PTQ.

Our discussion of P-inducers in the present and the following section is restricted to *simple sentences*. A simple sentence is defined as a declarative sentence (radical) that is not built up from component sentences by means of the English counterparts of the logical connectives '$\vee$', '$\wedge$', and '$\rightarrow$', and that does not

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Again, I am inclined to think that **only** is not a P-inducer, but should instead be translated into a rather complex semantic representation. Consider for example (iv)

iv) Only Bill has a car.

I agree with a proposal by Groenendijk and Stokhof, who translate (iv) roughly as 'For all x, if x has a car, then x = Bill'.

contain any of the so called intensional verbs like **believe, hope, seek,** etc. The present limitation to simple sentences will be motivated when we discuss the phenomenon of presupposition canceling in sections 2.1—2.5.

The first type of noun phrase to be tested is **every** + noun. The sentence (13):

13) John kissed every girl at the party

entails (14):

14) There were girls at the party

We can represent (13) roughly as

13′) Λx [girl a.t.p. (x)→kiss (j, x)]

The negation of (13) is

15) John didn't kiss every girl at the party

For (15) we can construct the usual scope ambiguity (with respect to negation), as indicated in

15a) $\sim \Lambda x$ [girl a.t.p. (x)→kiss (j, x)]
(i.e. 'It is not the case for every girl at the party that John kissed her')

versus

15b) Λx [girl a.t.p. (x)→$\sim$ kiss (j, x)]
(i.e. 'It is the case for every girl at the party that John didn't kiss her')

The ambiguity indicated in the logical transscriptions (15a) and (15b) does not correspond, however, to a *linguistic* ambiguity: the only intuitively possible reading of (15) is the *wide scope negation reading* represented in (15a). This phenomenon, which arises also with numerous other quantifiers (e.g. **the, some, any, all**) has been called a *scope restriction* (Hausser 1976).

For our present purposes it is sufficient to note that (15) *entails (14)* on the wide scope negation reading (which happens to be the only linguistically acceptable reading of (15)). Therefore part (b) of definition (2) (i.e. Strawson's definition of a semantic presupposition) is fulfilled in case of (15). Next we turn to the non-negated counterpart of (15), namely (13):

13) John kissed every girl at the party

To me, (13) intuitively clearly entails (14). I agree on this point with Edward Keenan (1972), for whom non-negated **every** as well as negated **every** leads to an entailment of existence.[4] Note in this connection the strangeness of example (16):

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[4] Keenan (1972) writes on page 422:
"Notice further that similar facts obtain for many uses of the universal quantifier **every.** Thus, (8a) presupposes (8c).

(8) a. Every soldier who protested was drunk.
b. Not every soldier who protested was drunk.
c. Some soldier protested

16) John kissed every girl at the party, you know there weren't any.

The fact that (16) is acceptable as an ironic statement or a joke does not affect our argument, because presuppositions may in general be canceled in such contexts. Note that

> John doesn't regret that Mary is pregnant, because Mary simply isn't pregnant.

is likewise acceptable under a certain ironic intonation. Yet 'regret' is an unchallenged example of what we call a P-inducer.

Since (13) as well as its wide scope negation (represented as (15b)) entail (14), we conclude that both (13) and (15) *presuppose* (14) according to Strawson's definition. It remains to show that it is in fact the occurrence of **every** in (13) and (15) which *induces* presupposition (14). Consider to this purpose the following two examples:

17) John kissed *a girl* at the party
18) John didn't kiss *a girl* at the party

(17) and (18) differ from (13) and (15) only insofar as the noun phrase **every girl** is replaced by **a girl.** We want to show now that neither (17) nor (18) presuppose (14) and that therefore the indefinite article—in contrast to **every**—is not a P-inducer. To this purpose it is sufficient to show that the *wide scope negation reading* of (18) does *not* entail (14). (18) is ambiguous between a so-called *specific* reading and a *non-specific* reading. The specific reading of (18) is represented as the narrow scope negation reading:

18a $\bigvee x[\text{girl a.t.p.}(x) \wedge \sim \text{kiss}(j, x)]$
(i.e. 'There is a particular girl at the party, and John didn't kiss her')

The non-specific reading of (18), on the other hand, corresponds to the wide scope negation reading:

18b) $\sim \bigvee x[\text{girl a.t.p.}(x) \wedge \text{kiss}(j, x)]$
(i.e. 'There is no girl at the party which John kissed')

Note that Strawson's definition refers only to wide scope negation (i.e. to sentences of the form $\sim$ A). This is in case of (18) the non-specific reading. Since it is one of the defining properties of the non-specific reading that it does not entail existence we conclude that (18) does not presuppose (14). Consequently, the

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Clearly (8a) entails (8c), and so does (8b) since it entails in particular that there were some soldiers that protested who were not drunk. So if (8c) is not true in some situation, then neither (8a) nor (8b) is true, so (8a) is not naturally true or false. Indeed, intuitively, if there were no soldiers who protested then it would not make sense to say that each one of them was drunk."

indefinite article cannot be an existential P-inducer. This conclusion is further supported by the fact that examples like (19) are perfectly acceptable on a normal, non-ironic reading:

19) John didn't kiss a girl at the party—in fact there weren't any.

After demonstrating that **every** is an existential P-inducer while the indefinite article is not, let us turn next to the quantifier **some.** It is clear that (20)

20) John kissed some girls at the party

as well as (21)

21) John didn't kiss some girls at the party.

both entail (14):

14) There were girls at the party.

The latter entailment is not conclusive, however, because of a scope restriction on **some:** (21) has intuitively only the narrow negation scope reading, as paraphrased in

22) There were some girls at the party who John didn't kiss.

Since it is in general true for English that *some* never occurs in simple sentences with wider scope than negation, we lack the linguistic data to decide on the basis of definition (2) whether *some* is an existential P-inducer. There is, however, an indirect argument according to which *some* is an existential P-inducer. The argument is based on the synonymy of *some not* and *not every*. Compare in this regard (23) and (24):

23) Some girls at the party weren't kissed by John.

24) Not every girl at the party was kissed by John.

The intuitive synonymy of **some not** and **not every** is paralleled by the logical equivalence

25) $\mathrm{V}x\,[f(x) \wedge \sim g(x)] \equiv \sim \Lambda x\,[f(x) \rightarrow g(x)]$

The quantifier **any,** in contrast to **some** and **every,** is not an existential P-inducer. This is sufficiently demonstrated by the following example, which does not entail that there were girls at the party:

26) John didn't kiss any girls at the party.

Our conclusion that **some** is an existential P-inducer while **any** is not is in conflict with Klima's proposal to derive **any** from **some** (Klima 1964). That is, a switch from **some** to **any** is rather implausible in light of our results. Our position is in agreement with Robin Lakoff (1969 b), who argued that "The distribution

of **some** and **any** depends not merely on relatively superficial syntactic information (negatives, questions, etc.), but also on presuppositions which may have no overt reflex" (p. 115). For an integrated formal account of the scope and occurrence restrictions of **any** within the framework of PTQ see Hausser, forthcoming.

After this investigation of the quantifiers **every, a(n), some,** and **any,** let us turn to the question of how these different quantifiers should be represented logically. The traditional representation of both **some** + noun and **a(n)** + noun (or, in the case of plural, $\emptyset$ + noun) in symbolic logic is a formula involving a non-restricted existential quantifier, such as

27) $\mathrm{V}x[f(x) \wedge g(x)]$

The representation of both, **every** + noun and **any** + noun, on the other hand, is a formula involving a non-restricted universal quantifier, such as (28) (e.g. Quine 1960):

28) $\Lambda x[f(x) \rightarrow g(x)]$

These traditional representations are unsatisfactory, however, in light of the fact that **some** and **every** induce existential presuppositions while **a(n)** and **any** do not. In order to distinguish in the logical language between those terms that are existential P-inducers and those that are not, I propose that in addition to the so called non-restricted quantification we should employ a second type of quantification—*restricted quantification*—which lends itself to interpretation as an existential presupposition inducer. Restricted quantification will be formally represented as in (29) and (30), where $x3[f(x)]$ may be read as 'x such that f(x)':

29) $\mathrm{V}x_3[f(x)]\, g(x)$

30) $\mathrm{V}x_3[f(x)]\, g(x)$

As truth-definition for restricted quantification let us assume that an elementary formula like (29) or (30) is without truthvalue with respect to a point of reference *i* if there is no x for which f(x) holds in *i*. Otherwise (29) and (30) have the same truthvalues as the corresponding formulas with non-restricted quantification, i.e. (27) and (28), respectively. The exact truthdefinitions for restricted quantification are stated in section 3.1 as part of our extension of PTQ.[5]

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5 A connection between restricted quantification and presuppositional phenomena has been suggested before. Kleene, for example, introduced a three valued logic in 1938 ("On a Notation for Ordinal Numbers", the Journal of Symbolic Logic, vol. 3, pp. 150—155). N. Rescher comments on this system as follows: "Kleene motivated the construction of these truthtables in terms of a mathematical application.He has in mind the case of a mathematical predicate P (i.e. a propositional function) of a variable x ranging over a domain D where "P(x)" is defined for only a part of this domain" (N. Rescher (1969), page 34).

By representing a term like **every girl** logically as V_{x3} [girl(x)] we insure that an example like (13)

13) John kissed every girl at the party

will be truthvalueless with respect to a possible world *i* if no girls at the party exist in *i*. All quantifiers which have been determined to be existential P-inducers will from now on be represented by means of restricted quantification.

1.4. *A preliminary analysis of the definite article*

The use of restricted quantification as a formal means to implement existential P-inducers can be made not only the basis for an intuitively satisfying representation of **every,** but it also permits a unified treatment of the singular and plural of the *definite article.* In connection with Russell's account of the singular definite article (discussed in section 1.1 above), it is of interest to note that the *plural* of the definite article refers to *all* individuals to which the definite description applies. Thus (31)

31) John kissed the girls at the party

does not mean that John kissed *some* of the individuals in the definite set of girls referred to by the definite description, but that he kissed *all* of them. I assume that this definite set of girls is implicitly specified over the context. For an analysis of definiteness in indexical terms see Hausser (1974b, chapter V). Furthermore, it can be observed that the negation of (31), namely

32) John didn't kiss the girls at the party

Nuel Belnap in his paper "Conditional Assertation and Restricted Quantification" arrives at restricted universal and existential quantification over a definition of conditional assertion which follows Quine and Rhinelander:

(1) If A is true then what (A/B) [= 'If A then B'] asserts is what B asserts. If A is false, then A/B is non-assertive.

Belnap analyzes the sentence "All crows are black" as

(11) Part 1. $\Lambda x(Cx/Bx)$ is assertive$_w$ just in case $VxCx$ is true$_w$.
Part 2. $(\Lambda x(Cx/Bx))_w$ is the conjunction of all the propositions $(Bt)_w$ such that Ct is true$_w$.

(Where "Ct" stands for 't is a crow' and "Bt" stands for 't is black'. The formulas in (11) are assertive or true with respect to a possible world *w*). Belnap does not mention the notion 'existential presupposition' anywhere in his paper. Nevertheless, his intentions obviously go in the same direction as ours. He arrives at restricted quantification in order to analyze a sentence like "All crows are black" as being *assertive* only if crows exist. I am grateful to Jack Murphy (Department of Philosophy, The University of Texas at Austin) who in the fall of 1972 drew my attention to the possibility of interpreting restricted quantification as an existential P-inducer.

is not ambiguous (with respect to the scope of negation). (32) can intuitively only mean that John kissed *none* of the girls in question (and not that he failed to kiss some of them). Thus the only possible reading of (32) is the narrow scope negation reading (under the assumption that **the** is represented by means of a universal quantifier).

The above conclusions regarding the plural of **the** are in conflict with the basic assumptions Russell made in his representation of the singular definite article:

i) Russell represents the singular definite article by means of the existential quantifier. In light of the evidence for representing plural-**the** by means of a universal quantifier, it seems highly dubious that singular and plural of one linguistic quantifier word should be represented by two different logical quantifiers.

ii) It is crucial for Russell's analysis that both narrow and wide scope negation is possible in connection with singuar-**the.** Yet we saw that plural-**the** in interaction with negation is interpreted only in the sense of narrow scope negation.[6]

What is Russell's reason for using an existential rather than a universal quantifier? The answer is that in non-restricted quantification only a (non-negated) existential quantifier renders the desired existential entailment. Thus, even though

33) $\mathrm{V}x[\Lambda y[\mathrm{king}(y) \longleftrightarrow y = x] \wedge \mathrm{bald}(x)]$

and

34) $\Lambda x[\Lambda y\, \mathrm{king}(y) \longleftrightarrow y = x] \rightarrow \mathrm{bald}(x)]$

refer both to exactly one king, it is only (33) which *entails the existence* of a unique king.

This difference is not present, however, in the corresponding formulas using *restricted* quantification:

35) $\mathrm{V}x3\,[\Lambda y[\mathrm{king}(y) \longleftrightarrow y = x]]\ \mathrm{bald}(x)$

36) $\Lambda x3\,[\Lambda y[\mathrm{king}(y) \longleftrightarrow y = x]]\ \mathrm{bald}(x)$

The simple proof that (35) and (36) are logically equivalent is left to the reader.

Since (35) and (36) are equivalent in our extension we can represent the singular as well as the plural of **the** uniformly by means of *a restricted universal quantifier*. Singular and plural-**the** are similar insofar as they both *presuppose existence* and *take wider scope than negation.* The only difference between singular and plural-**the** is a numerical one: singular-**the** presupposes the existence of exactly

6 The scope restrictions of **the** regarding negation are subject to the general conditions on scope restrictions of *presupposing quantifiers,* which are developed in Hausser (1976b).

one individual, while plural-**the** presupposes the existence of at least two individuals (c.f. Hausser 1974b, chapter II).

Our argument for postulating P-inducers is based on straight-forward substitution tests. We demonstrated that sentences which differ only in that they contain different quantifiers have different presuppositions and entailments. More specifically, in order to show that a quantifier *q* is not an existential P-inducer it was sufficient to show that a simple sentence A containing *q* does not entail the existence of *q* + *noun, or* that the negation of A (no matter whether it is wide or narrow scope negation) does not entail the existence of *q* + *noun*. In this way it is easy to show that in addition to the indefinite article and **any** the natural numbers **one, two, three,** etc. are not existential P-inducers. On the other hand, in order to show that a quantifier *p* is an existential P-inducer, we had to show that a simple sentence A' containing *p* does entail the existence of *p* + *noun* and, furthermore, that the wide scope negation of A' entails the existence of *p* + *noun*. We demonstrated that **every** is an existential P-inducer, and it is easy to show that the quantifier **both** is likewise an existential P-inducer. In the case of **some** and **the,** however, we have to make use of indirect methods because **some** and **the** are subject to interfering scope restrictions regarding negation.

Our substitution tests (which are a fundamental tool of linguistic analysis) showed clearly that it is *sentences* which have presuppositions. This is not to deny, of course, that speaker and hearer are very likely to have assumptions about the world. The linguistically significant point, however, is that sentences have semantic properties the speaker has to reckon with if he wants to use a sentence successfully. Success depends on ones intentions. For example, if a speaker wants to be sincere, he should take care that the semantic presuppositions (if any) of the sentence he uses are not in conflict with his assumptions. If the speaker wants to mislead or make a joke, on the other hand, he may choose a sentence the semantic presuppositions of which are contrary to his beliefs. Whether a sentence is used sincerely, as a joke, or as a misleading statement can be evaluated in a model-semantic way by interpreting the sentence with respect to the possible world representing the speaker's beliefs.

2. The Presuppositions of Complex Sentences

2.1. *The projection problem*

Linguistic intuition is the basis for deciding whether a certain lexical item is a presupposition inducer or not. Once the set of P-inducers of a natural language is established, however, we are enabled to systematically compute the relevant presuppositions of any *simple* sentence without depending on intuition. Similarily in the case of complex sentences: in order to find out which conditions cause the *canceling* of P-induced presuppositions in certain complex sentences we

depend on linguistic intuition. As soon as we have described systematically under which conditions component presuppositions fail to be presupposed by a complex sentence, however, we are enabled to predict the relevant presuppositions of any given sentence, be it simple or complex.

In the following let us investigate a case of presupposition canceling which arises in connection with complex sentences of a certain kind. That is, complex sentences which are composed from simple sentences by means of the connectives **and, or,** and **if . . . then.** We are interested in such sentences because **and, or,** and **if . . . then** are the English counterparts of the logical connectives '$\wedge$', '$\vee$', and '$\rightarrow$'. Since the usual two-valued definition of the logical connectives is not appropriate in the presuppositional calculus we aim to develop, we must choose a non-classical definition of the logical connectives. Our choice will be guided by the condition that the non-classically defined logical connectives should *adequately reflect the intuitive canceling and non-canceling* of component presuppositions in complex sentences built up by means of **and, or,** and **if . . . then.** Before we turn to the technical details, however, let us discuss the linguistic facts and the linguistic literature which first described these facts.

Langendoen and Savin (1971) directed attention to what they called the "projection problem for presuppositions", i.e. the question of which of the presuppositions of the component sentences are among the presuppositions of the compound sentence. Consider for example (37):

37) If Fred has stopped beating Zelda, then Fred no longer resents Zelda's infidelity.

(37) consists of the component sentences (38) and (39):

38) Fred has stopped beating Zelda.

39) Fred no longer resents Zelda's infidelity.

Component sentence (38) presupposes (38A):

38A) Fred used to beat Zelda.

Component sentence (39), furthermore, presupposes (39A):

39A) Zelda was unfaithful.

The projection problem regarding example (37) is the question: does (37) share the presuppositions of its components, e.g. (38A) and (39A)?

In the case of our particular example one is intuitively inclined to answer this question with 'yes'. Based on examples like (37), Langendoen and Savin proposed the hypothesis that the presuppositions of compound sentences are simply the logical sum of the presuppositions of their components. Jerry Morgan (1969) called this assumption the *cumulative hypothesis*. Morgan noted,

however, that the cumulative hypothesis is not strictly valid. As an exception he cites:

40) If Jack has children, then all of Jack's children are bald

Here the second component sentence, (41),

41) All of Jacks children are bald

presupposes the sentence

42) Jack has children.

Yet the compound sentence (40) does not presuppose (42) intuitively. In other words, (40) does not share a presupposition of one of its components.

It was Lauri Karttunen who in a series of papers (1973a, 1974b, 1974a) proposed a systematic account for such exceptions to the cumulative hypothesis as exemplified by (40). Karttunen compared (40) with (43):

43) If baldness is hereditary, then all of Jack's children are bald.

In case of (43), Karttunen observes that the presupposition of the second component (which is identical with (42)) is also a presupposition of the whole complex sentence. In other words, (43) seems to presuppose (42), while (40) does not.

This phenomenon, that in the case of two syntactically similar sentences one shares a presupposition of a component sentence while the other does not, has been called by Karttunen (1973a) the "filtering of a presupposition". Filtering arises in connection with the connective **if ... then, and,** and **or.** An illustration of the filter effect of **and** and **or** are the sentences (44) and (45):

44) Jack has children, and all of Jack's children are bald.

45) Either Jack has no children or all of Jack's children are bald.

According to Karttunen, neither (44) nor (45) presuppose (42). In other words, (44) and (45) *filter* a presupposition of their second component sentence.

Besides the filters, Karttunen (1973a) discusses verbs like **believe, hope,** and **say,** which he calls "plugs". These verbs seem to prevent presuppositions of their component sentence from being among the presuppositions of the complex sentence. Thirdly, Karttunen postulates a class of verbs which he calls "holes" because they seem to 'let through' the component presuppositions of their complement (e.g. **know, regret, surprise,** etc.). In later papers Karttunen refined his hypotheses about filters, plugs and holes. However, the general conceptual approach underlying the filters, plugs, and holes remained unchanged throughout the later refinements, as will be shown below on the example of "filters".

2.2. *Filter conditions as solution to the projection problem*

In order to describe the phenomenon of filtering, Karttunen (1973a) proposed his so-called *filter conditions*, which are stated under (46), (47), and (48) below:

46) Let S stand for any sentence of the form "If A then B".
   (a) If A presupposes C ($A \gg C$), then S presupposes C ($S \gg C$).
   (b) If B presupposes C ($B \gg C$), then S presupposes S ($S \gg C$) unless A semantically entails C ($A > C$).

47) Let S stand for any sentence of the form "A and B".
   (a) If $A \gg C$, then $S \gg C$
   (b) If $B \gg C$, then $S \gg C$ unless $A > C$.

48) Let S stand for any sentence of the form "A or B".
   (a) If $A \gg C$, then $S \gg C$.
   (b) If $B \gg C$, then $S \gg C$ unless $\sim A > C$.

The refined versions of these filter conditions presented in Karttunen (1973b, 1974a) differ from (46—48) only in that a *context of use* is taken into account. (46), for instance, is rephrased in Karttunen (1974a, p. 183) as follows:

46*) The presuppositions of "If A then B" (with respect to context X)[7] consists of

   (i) all of the presuppositions of A (with respect to X) and
   (ii) all of the presuppositions of B (with respect to $X \cup A$) except for those entailed by the set $X \cup A$ and not entailed by X alone.

It is obvious that the inclusion of a context in (46*) does not alter Karttunen's empirical assumptions: in (46) as well as in (46*) the presuppositions of an A-sentence may never be filtered, while a presupposition of a B-sentence is filtered only if it is entailed by A.

In the following, I will refer to the version of the filter conditions presented by Karttunen (1973a). Since the later versions in Karttunen (1973b and 1974a) share the indicated empirical assumptions of Karttunen (1973a), the results of our discussion apply also to the later versions. Karttunen's account is of interest to us not only because it is at present the most extensive linguistic attempt to describe the presuppositions of complex sentences, but also because Karttunen has repeatedly justified his approach by the alleged failure of a semantic approach to account for the linguistic phenomenon of "filtering".

It is easy to demonstrate that the filter conditions (46)—(48) account, in an intuitive sense, for the examples (37—45). (40), for example, can be analyzed according to condition (46) as follows:

49) A = Jack has children.
   B = All of Jack's children are bald.
   C (where $B \gg C$) = Jack has children

Obviously, A semantically entails C; A and C are actually identical! Therefore, C is filtered according to (46).

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7 Where *context* is defined as a set of propositions.

(43), on the other hand, is analyzed as follows:

50) A = Baldness is hereditary.
B = All of Jack's children are bald.
C (where B ≫ C) = Jack has children.

Here A does not entail C; therefore C is not filtered according to (46).

Since the filter conditions capture the intuitive facts in a number of cases, we are interested in the question: what is the explanatory power of this analysis. On the one hand, the filter conditions make concrete predictions (e.g., the presuppositions of an A-sentence may never be filtered). On the other hand, assuming that such predictions are correct, there should be some kind of explanation for them; but explanations are not to be found in Karttunen (1973a, 1973b, 1974a).

2.3. *A semantic solution to the projection problem*

As an alternative account to the filter phenomenon let us consider a semantic treatment. Such an approach requires (a) a semantic notion of presupposition, and (b) a suitable propositional logic. For (a) I propose the use of Strawson's definition, which we already discussed in section 1.1. For easier reference it is restated below:

51) S presupposes C if and only if:
(a) if S is true, then C is true,
(b) if not-S is true, then C is true.

This definition is equivalent to (52):

52) S presupposes C if and only if S is truthvalueless unless C is true.

According to Strawson's definition a sentence is truthvalueless if (but not 'if and only if'!!) one of its presuppositions is not fulfilled.

I turn now to the discussion of a suitable sentence logic. Since a sentence doesn't always need to have a truthvalue (according to Strawson's definition), the usual two-valued propositional calculus cannot suffice for our purpose. Instead I propose the choice of Kleene's three-valued system (c.f. N. Rescher (1969), p. 34f.). In this system the logical connectives are defined as in (53):

53)

A	~A
1	0
0	1
#	#

∧	1	0	#
1	1	0	#
0	0	0	0
#	#	0	#

∨	1	0	#
1	1	1	1
0	1	0	#
#	1	#	#

→	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

↔	1	0	#
1	1	0	#
0	0	1	#
#	#	#	#

Kleene's system is a presuppositional logic of the so called "strong type". The crucial feature of a strong type logic is that it assigns bivalent truthvalues (i.e. either 1 or 0) to complex sentences even in cases where one component sentence is # (where '#' represents for us an undefined rather than a third truthvalue). Consider for example the definition of '∨': if 'A' is 1 and 'B' is #, then 'A∨B' is 1 according to (53). The intuitive motivation is that the truth of one disjunct is sufficient to make the whole disjunction true. However, if 'A' is 0 and 'B' is #, then 'A∨B' is #.

It is solely the characteristic truthvalue assignment of a strong type logic which is of interest for our semantic analysis of filtering. Thus, instead of Kleene's system we could as well have chosen Łukasiewicz' or Van Fraassen's system, which are likewise of the strong type (c.f. N. Rescher (1969), p. 22f., and Van Fraassen (1968, 1969)). The systems of Kleene, Łukasiewicz, and Van Fraassen agree in their truthvalue assignment *except* in cases where both component sentences are #. The cause for disagreement in these particular cases lies in the complicated question of whether classical tautologies like 'A∨∼A' and contradictions like 'A ∧ ∼A' should be valid if A is #. In Kleene's system this question is answered with an unconditional 'no'. I have chosen Kleene's system among the various strong type logics because there is linguistic evidence corroborating Kleene's assumption regarding classical tautologies and contradictions. Consider for example:

Either John regrets that the democrats lost or he doesn't.

This sentence has the structure of a classical tautology, yet intuitively it presupposes that the democrats lost (i.e. it should be truthvalueless if the democrats did in fact win). Note, however, that the question concerning the validity of classical tautologies and contradictions is a separate issue: our decision in this respect has no consequence for our semantic analysis of filtering, which will be based on the most general properties of strong type logics.

In contrast to a strong type logic, a logic is of the *weak* type if it assigns # to 'A ∧ B', 'A ∨ B', and 'A → B' whenever 'A' or 'B' is #. As Lauri Karttunen (1973a) points out, a weak type logic is equivalent to the cumulative hypothesis: the systematic exceptions to the hypothesis described by means of the filter-conditions, for example, could not be accounted for in a weak type logic. Therefore, a weak type logic is linguistically inadequate.

Let us return now to the phenomenon of filtering. Our goal is to explain this intuitive phenomenon truthfunctionally on the basis of a strong type logic. We assume that '∧', '∨', and '→', as defined in (53), are reasonably close semantic representations of English **and, or**[8], and **if . . . then.** This assumption is simplistic

[8] The definition of '∨' in (53) represents the inclusive "or" in which the alternative in which both disjuncts are true is not meant to be excluded. In English, "or" is used both in the inclusive and the exclusive sense. Examples for the inclusive and exclusive use of "or" are given in Thomason (1970, p. 32, 33):

in certain respects, but our analysis of filtering to be given belows is based on such basic semantic features, that a more refined analysis of the connectives (in terms not only of truthvalues, but also in terms of correctness conditions) would be compatible with the principle of our analysis.

In order to demonstrate the basic idea of our semantic analysis of the filter phenomenon, let us consider once more example (40):

40) If Jack has children, then all of Jack's children are bald.

It is a simple structural fact of sentence (40) that the presupposition of the B-sentence (i.e. "Jack has children") is fulfilled, whenever the A-sentence is true. If, however, the A-sentence is 0, then it does not matter whether the B-sentence is 1, 0 or #— at least not in a strong type logic. The structural interdependence of truthvalues in sentence (40) is expressed in truthtable (54):

54) (where B ≫ C)

A	B	C	A→B
1	1	1	1
1	0	1	0
0	#	0	1

The rightmost row of (54) shows all possible truthvalues of (40). The crucial result is that sentence (40) can *never lack a truthvalue* according to our semantic analysis: it is always either true or false. It is this semantic property of (40) (caused by the inner structure of the example) which explains why the presupposition is intuitively filtered. The presupposition seems filtered, *because its failure never results in truthvaluelessness* of (40).

In comparison consider now sentence (43):

43) If baldness is hereditary, then all of Jack's children are bald.

(43), according to Karttunen, does not filter the presupposition of the B-sentence (i.e. "Jack has children"). The truthtable corresponding to (43) is (55):

"For instance, if Smith maintains that Jones is a miser or a vindictive troublemaker, we would not say that Smith was wrong in case both in fact were true, or he would not have made his point so guardedly; he would have said that Jones was a miser *and* a vindictive troublemaker. ... But we ought to be a bit hesitant about translating something like 'My son will mow the lawn or I'll take way his teddy bear' in this way, since it connotes that it will not be the case that both disjuncts hold true.
There are many examples in which the joint case simply does not come up: for instance, 'He is in Boston or Zagreb', or 'Either he beats his wife or he doesn't'. In both of these, the situation in which both disjuncts are true is excluded out of hand, and it doesn't seem a matter of great importance whether or not we say these sentences are true in case both disjuncts are."

The examples involving "or" in this paper are of the kind in which the joint case doesn't come up.

55) (where B ≫ C)

	A	B	C	A→B
(a)	1	1	1	1
(b)	0	1	1	0
(c)	1	0	1	0
(d)	0	0	1	1
(e)	1	#	0	#
(f)	1	#	0	1

(55) shows that (43) *may be truthvalueless* if the presupposition of the B-sentence is not fulfilled. According to my hypothesis, it is this *possible* influence on the truthvalue of the whole sentence that explains why the B-sentence-presupposition in (43) does not seem to be filtered.

In short, according to our semantic analysis the intuitive phenomenon of filtering is explained over *possible* influence of a component presupposition on the truthvalue of the whole: if a component presupposition has no possible influence on the truthvalue of the whole, then the component presupposition seems intuitively to be filtered; on the other hand, if a component presupposition does have a possible influence on the truthvalue of the whole (as in (55)), then it *appears* not to be filtered. Exactly the same happens in case of the other connectives **and** and **or.** Thus,

45) Either Jack doesn't have children or all of Jack's children are bald.

will always be bivalent according to our semantic representation, while

56) Either baldness is not hereditary or all of Jack's children are bald.

may be truthvalueless given that Jack has no children.

Our semantic approach leads to the same results as the filter conditions with respect to the examples considered by Karttunen. Instead of saying that a component presupposition C is filtered in S under certain conditions, we say that C has no influence on the truthvalue of S. Theoretically, however, the two approaches have quite different status. The filter conditions are additional machinery in the grammar that does not relate the described phenomena to any more general principles. The semantic analysis, on the other hand, is based on the truth definitions of the logical connectives which in one form or another have to be part of the grammar anyway.

2.4. *Alleged arguments against the semantic analysis*

Karttunen prefers filters over a semantic approach, because, in his opinion, semantic systems of the strong type have a serious flaw that makes them unacceptable from a linguistic point of view. This opinion is based on examples like (57) and (58):

57) Paris is the capital of France, and the king of France is bald.

58) Marseille is the capital of France, and the king of France is bald.

Karttunen (1973a, p. 188) argues with respect to these examples as follows:

> "Assuming that the facts are as we know them to be, in Łukasiewicz' and Van Fraassen's system (35a) (i.e. our 57, R.H.) presupposes that France has a king, since the sentence is neither true nor false in case the king does not exist. On the other hand, given the actual state of affairs, (35b) (i.e. our 58, R.H.) in their logics does not presuppose the existence of the king, since the falsehood of the first conjunct is sufficient to make the conjunction bivalent. From the view of ordinary language, this outcome is definitely unacceptable. Relative to our actual world, where the forms of government a country has are not determined by the choice of the capital, both sentences surely presuppose that France has a king."

Does this argument really hold? Let us have a look at the truthtables of (57) and (58). Since (57) and (58) have exactly the same syntactic structure we can represent them by the same truthtable, namely (59):

59)

	A	B	C	A ∧ B
(a)	1	1	1	1
(b)	1	0	1	0
(c)	1	#	0	#
(d)	0	1	1	0
(e)	0	0	1	0
(f)	0	#	0	0

Where A = {Paris / Marseille} is the capital of France
B = The king of France is bald
C = The king of France exists

(59)c) represents the case of (57) being interpreted with respect to the present actual world (i.e. A is 1 and B is #). (59f), on the other hand, represents the case of *(58)* being interpreted with respect to the present actual world (i.e. A is 0 and B is #). We may, however, interpret (57) and (58) also with respect to possible worlds other than the real world; a possible world, for example, in which Marseille rather than Paris happens to be the capital of France. In that case (57) would be 0 while (58) would be #. In short, contrary to Karttunen's claim, according to a strong type logic there is no semantic difference between (57) and (58).

The crucial point is that neither (57) nor (58) presuppose C (= "there exists a king of France"). The reason is that according to Strawson's definition a

sentence S presupposes a sentence C only if S is truthvalueless *whenever* C is not fulfilled (c.f. definition (52) above). This, however, is not the case in (59). In (59f) the component presupposition C is not fulfilled, yet the complex sentence is 0 (i.e. is bivalent). Therefore, C cannot be a presupposition of either (57) or (58). Instead, C is a *component presupposition* that has an *influence* on the truthvalue of the whole sentence (without being a presupposition of the whole sentence). According to our hypothesis, component presuppositions which have a possible influence on the truthvalue of the whole appear intuitively not to be filtered.

In Karttunen (1973b) we find another argument against the semantic analysis of the projection problem, based on the following example (originally thought up by Thomason):

60) Today is Wednesday or my roof found a hole in its pocket.

Karttunen comments:

> "As Thomason observes, (7) (i.e. our (60), R.H.) is unacceptable on any day of the week, and this can be traced back to the fact that the second disjunct is a sentence that suffers from a chronic failure of presupposition. It is disillusioning to find out that Van Fraassen's theory, which Thomason supports, cannot in principle do the job. Instead it yields a very bizarre result. In Van Fraassen's system (7) has the value *true* on Wednesdays and no truthvalue on other days of the week. . . .
>
> Those who have written about presuppositions have in general assumed that a sentence suffers from lack of truthvalue just in case it has a failing presupposition. What else should be there to make it non-bivalent? But since (7) is supposed to be true on Wednesdays, it does not have the failing presupposition every day. The semantic theory Thomason supports commits him to the view that presuppositions of compound sentences can vary depending on the contingent features of a given situation."
>
> (Karttunen 1973b, p. 4)

Karttunen fails to distinguish here between component presuppositions which have an influence on the truthvalue of the whole, on the one hand, and component presuppositions which are really presupposed by the whole sentence, on the other. Thus he arrives at the mistaken assumption that a sentence without truthvalue must have a failing presupposition (in reality a failing component presupposition may lead to a truthvalueless complex sentence—this is the characteristic property of logics of the strong type). Consequently, it is not the presuppositions of sentence (60) which vary depending on the day of the week, but only the truthvalue—and that's just how it should be. Finally, what is the presupposition of (60 A) really supposed to be?

60A) My roof found a hole in its pocket

If Karttunen assumes a failing presupposition in case of (60A) he should state what exactly this failing presupposition is—especially if it is the novelty of a "chronically failing" presupposition. To me (60A) is simply nonsense and I cannot agree with Karttunen's (pragmatic) notion of presupposition if it does not distinguish nonsense from P-induced presuppositions.

2.5. *A counterexample to the filter conditions*

We have argued that the filter conditions are additional machinery which does not relate the described phenomena to any general principle of the grammar. Futhermore, we have shown that the alleged advantages of the filter conditions over a semantic approach are based on a misunderstanding of the semantic approach. It remains to compare the filter conditions and the semantic approach empirically.

It is obvious that the two approaches make different linguistic predictions. We have already noted that the filter conditions in principle do not filter A-sentence-presuppositions (c.f. (46)—(48) above). The connectives defined in (53), on the other hand, are symmetrical. Let us therefore look for linguistic examples that provide evidence for or against the different predictions of the different approaches. In this regard consider (61):

61) The liquid in this tank has either stopped fermenting or it has not yet begun to ferment.

Assume for example that the liquid ferments at a certain temperature and the thermometer on the tank indicates that the liquid is not fermenting. The disjunction of (61) consists of two component sentences, (62A) and (62B):

62A) The liquid stopped fermenting.
62B) The liquid has not yet begun to ferment.

Component sentence (62A) presupposes (62C):

62C) In the past, the liquid was fermenting.

Component sentence (62B), on the other hand, presupposes the contrary of (62C), namely (~62C):

~62C) In the past, the liquid was *not* fermenting.

Intuitively, it is clear that (62) as a whole presupposes neither (62C) nor (~62C). Rather, (61) as a whole entails (62D):

62D) Presently, the liquid is not fermenting.

Karttunen's filter conditions fail in case of example (61).

The A-sentence-presupposition, (62C), cannot be filtered since the filter conditions in general do not provide for the filtering of any presuppositions of an A-sentence. The B-sentence presupposition, (~62C), on the other hand, could be filtered only if it were entailed by (62A), but this is not the case. In fact, (62A) and (~62C) are incompatible with each other. Obviously, (61) is not an isolated counterexample to the filter conditions in their various formulations, but represents a whole class of (rather special) sentences.

Let us see now how the semantic approach handles a sentence like (61). The following truthtable indicates the semantic interaction between (62A), (62B), (62C) and (~62C):

63)

	C	~C	A	B	A∨B
(a)	1	0	1	#	1
(b)	1	0	0	#	#
(c)	0	1	#	1	1
(d)	0	1	#	0	#

The resulting truthvalues are in concord with our intuition. If the liquid is not fermenting in the situation with respect to which we interpret (61) (i.e. if either (62A) or (62B) is true), then the whole disjunction is true. If, however, the liquid is fermenting, then the whole disjunction is truthvalueless. Both (62C) and (~62C) are component presuppositions with an influence on the truthvalue of the compound sentence, but which are not presupposed by the compound sentence. It is because of the failure of (62C) and (~62C), respectively, that the resulting truthvalue in (63b) and (63d) is # rather than 0. Note that (62D) is an entailment rather than a presupposition of (61), because the negation of (61), i.e. (64):

64) It is not the case that the liquid either stopped fermenting or hasn't begun to ferment.

entails the contrary of (62D), namely

65) Presently, the liquid is fermenting.[9]

Note that in order to show the inadequacy of the filter approach up to Karttunen (1974a) the following (simpler) example would suffice:

[9] In Hausser (1975), I assumed that (62D) is presupposed by (62). This assumption was based on the combination of truthvalues: The complex sentence is bivalent if D is true, but # if D is false. Theo Vennemann and Dietmar Zaefferer convinced me, however, that this is intuitively incorrect (see (64) and (65)). Since (62D) is a presupposition of (62) according to definition (52), but not according to definition (51), it seems that (51) and (52) are in fact not equivalent (contrary to Van Fraassen (1968), (1969)) in a presuppositional logic of the strong type.

66) All of Jack's children are bald, or Jack doesn't have children.

Since (66) does not presuppose that Jack has children it is a counterexample to the claim that A-sentence-presuppositions may never be "canceled".

One reason for the deficiency of the filter approach is in my opinion that it rests on a twofold distinction: a complex sentence either shares a presupposition of a component or it doesn't. In a logical system of the strong type, on the other hand, we have a fourfold distinction:

67a) Component presuppositions which have *no influence* on the truthvalue of the compound sentence (e.g. (42) in the case of (40), and (62C), (~62C) in the case of (61)).

67b) Component presuppositions which *have an influence* on the truthvalue of the compound sentence, but are still not presupposed by the compound sentence (e.g. (38A), (39A) in the case of (37), and (42) in the case of (43).

67c) Component presuppositions which are at the same time presuppositions of the compound sentence (and thus, by definition, have an influence on the truthvalue of the compound sentence).

67d) Presuppositions of a compound sentence which are not presupposed by any of its components.

One of the possibilities which we have not encountered so far, is that of (67c). This is the type of presupposition which according to the filter conditions (and the cumulative hypothesis) should be the standard case. Note that even the presuppositions (38A) and (39A) in example (37) are instances of (67b) rather than (67c)! Nevertheless, however, presuppositions of type (67c) do occur and thus constitute additional support for the fourfold distinction inherent in our semantic approach. Consider for example:

68) Bill regrets that John stopped dating Mary.

Here we have the component sentence

68B) John stopped dating Mary.

which presupposes

68C) John used to date Mary.

This component presupposition is also a presupposition of the whole sentence (68): (68) will be truthvalueless *whenever* (68C) is not fulfilled.

It is an interesting question whether presuppositions of type (67 d) occur in natural language. Consider for example (69):[10]

69) Q: Who in Nixon's cabinet lied?
A: Mitchell did.

If we interpret (69) with respect to the actual world, it has the truthvalue 1, while the question-answer pair (69*)

69*) Q: Who in Nixon's cabinet lied?
A: Mitchell didn't.

would have the value 0. The point of (69) and (69*) is that they *presuppose*

69C) Mitchell was a member of Nixon's cabinet.

In contrast consider

70) Q: Who in Nixon's cabinet lied?
A: Carl Albert $\{\begin{matrix}\text{did}\\ \text{didn't}\end{matrix}\}$.

which would be # with respect to the real world, because the presupposition

70C) Carl Albert was a member of Nixon's cabinet.

is not fulfilled. Since the presuppositions (69C) and (70C) are not presupposed by either the question alone or the answer alone (but arise through the syntactico-semantic interaction between question and answer), they could be classified as instances of type (67 d).

3. Implementing Presuppositions into PTQ

3.1. The treatment of existential presuppositions

In the previous sections we discussed the formal treatment of presuppositions on the assumption that the occurrence of (semantic) presuppositions is

[10] (69) is derived from a similar example in German in Marga Reis (1975, p. 20, (16) and (17)). Ms. Reis argues on the basis of this kind of example that presuppositions are *context-dependent*. In a narrow (and trivial) sense I agree with her: it is one of the hypotheses of this and earlier papers of mine that presuppositions may be canceled in certain well defined environments. In this sense presuppositions are context dependent. But this kind of context dependency is neither an argument for Stalnaker's notion of a pragmatic presupposition (which Marga Reis advocates) nor is it an argument against P-induced semantic presuppositions. In order to argue for pragmatic presuppositions the presuppositions must be shown to depend on situational contexts (c.f. example (11)), not on a specific syntactic environment. In case of (60) the presupposition in question arises on the basis of a very specific syntactic interaction, is thus predictable, and may be classified as another instance of a semantic presupposition.

systematically predictable. Two principles were assumed to be at work: on the one hand, in simple sentences presuppositions are induced by certain lexical items, which we called P-inducers. On the other hand, P-induced presuppositions may be canceled in certain contexts. We demonstrated the systematic nature of these two principles in detail on two representative examples: the concept of a P-inducer was exemplified in connection with quantifiers and existential presuppositions. The phenomenon of presupposition canceling in all its complexity was exemplified in connection with the connectives **and, or,** and **if ... then.** We showed why certain arguments against a semantic treatment of presuppositions do not hold, and that our semantic treatment based on a logic of the strong type does in fact account very well for the intuitive facts.

Our approach differs from earlier work in this area in that it systematically combines the principle of presupposition-induction and the principle of presupposition canceling. Since our main concern up to now was to develop a general approach we avoided a rigorous formal treatment. Thus important parts of the grammatical system with respect to which our analysis was designed were left unspecified. For example, we related sentences of English to formulas of the auxiliary language of first order predicate calculus on the basis of *intuition alone,* rather than *generating* the sentences of English by a finite set of rules, and providing a formal mechanism for *translating* sentences of English into the predicate calculus. Also, we did not specify formal models with respect to which the formulas of predicate calculus should be *interpreted.*

In order to overcome this incompleteness in the presentation of our treatment of presuppositions and in order to arrive at a more realistic treatment of quantifiers in Montague Grammar let us proceed now to incorporate our results regarding existential presuppositions into the fragment of English described in Montague's "Proper Treatment of Quantification in Ordinary English" (PTQ). Since a number of *existential P-inducers* (i.e. quantifiers) has been determined and a formalism for their logical representation (i.e. restricted quantification) has been developed, the implementation of existential presuppositions into PTQ will consist in a straightforward formal extension.

For our present purpose we can leave the categorial surface syntax of PTQ unchanged (c.f. '2. The Syntax of English', Montague 1973, pp. 222—228, in Hintikka et. al. (eds.)). Major changes of the syntax of English in PTQ would indeed be necessary if we wanted to incorporate quantifiers like **all, both,** the natural numbers greater than **one** as well as the plural of **the, some,** the indefinite article, etc. But in the present section our discussion will be limited to those quantifiers which are already treated in PTQ, namely **every,** the singular of **the,** and the singular of the indefinite article. In addition, we will provide translations for the quantifiers **some** and **any.** For a treatment of syntactic and semantic plural within the framework of PTQ see Hausser (1974b, chapter III and IV).

On the level of intensional logic we have to add the symbols representing restricted quantification to the set of *meaningful expressions* c.f. '3. Intensional Logic', Montague 1973, pp. 228—232). Thus we replace number (5) on page 229 by (*5):

71) (5*) If $\varphi, \psi \in ME_t$ and u is a variable then $\sim\varphi$, $[\varphi \wedge \psi]$, $[\varphi \vee \psi]$, $[\varphi \rightarrow \psi]$, $[\varphi \longleftrightarrow \psi]$, $Vu\varphi$, $\Lambda u\varphi$, $Vu\,\exists\,{}^{(}\varphi{}^{)}\psi$, $\Lambda u\,\exists\,{}^{(}\varphi{}^{)}\psi$, $\Box\varphi$, $W\varphi$, $H\varphi \in ME_t$.

Secondly, we have to provide for the possibility that a sentence may be 'truthvalueless'. Rather than introducing a third truthvalue, let us interpret *truthvalueless* in the sense that a truthvalue is not defined. In order to implement this interpretation of *truthvalueless* we have to change the definition of possible denotations of type $\langle a, b\rangle$ (c.f. Montague (1973), p. 230) by setting

72) $D_{\langle a, b\rangle, A, I, J} = \{$partial functions from $D_{a, A, I, J}$ to $D_{b, A, I, J}\}$

In order to treat existential P-inducers it would be sufficient to restrict partial functions to *propositions* (i.e. partial functions from $I \times J$ to $D_{t, A, I, J}$). Since we will define partial functions of various types in the next section, however, the generality of (72) is motivated.

Thirdly, we have to change a number of *truth definitions* in order to specify not only when a sentence is *true* or *false* but also when it is *truthvalueless*. On page 231 of Montague (1973) we replace the truth definitions (6) and (7) by (6*) and (7*), respectively.

73) *The truth definitions of the logical connectives*:
(6*) If $\varphi \in ME_t$ then $[\sim\varphi]^{a, i, j, g}$ is 1 iff $\varphi^{a, i, j, g}$ is 0 and $[\sim\varphi]^{a, i, j, g}$ is 0 iff $\varphi^{a, i, j, g}$ is 1. The truth definitions for $[\varphi \wedge \psi]^{a, i, j, g}$, $[\varphi \vee \psi]^{a, i, j, g}$, $[\varphi \rightarrow \psi]^{a, i, j, g}$ and $[\varphi \longleftrightarrow \psi]^{a, i, j, g}$ are as indicated in (74):

74)

$\varphi \wedge \psi$	1	0	#
1	1	0	#
0	0	0	0
#	#	0	#

$\varphi \vee \psi$	1	0	#
1	1	1	1
0	1	0	#
#	1	#	#

$\varphi \rightarrow \psi$	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

$\varphi \longleftrightarrow \psi$	1	0	#
1	1	0	#
0	0	0	#
#	#	#	#

According to (73), a truthvalue is defined for $[\sim\varphi]$ if and only if φ has a truthvalue. The truth definitions for the other connectives are specified in form of (74), because the truthtable method is more perspicuous. For example, the truthtable for '$\wedge$' is to be interpreted in the sense of the following definition:

75) If $\varphi, \psi \in ME_t$, then $[\varphi \wedge \psi]^{a, i, j, g}$ is 1 iff $\varphi^{a, i, j, g}$ and $\psi^{a, i, j, g}$ is 1. $[\varphi \wedge \psi]^{a, i, j, g}$ is 0, iff $\varphi^{a, i, j, g}$ or $\psi^{a, i, j, g}$ is 0 (otherwise a value is not defined).

We followed Kleene in the definition of the logical connectives, for the reasons indicated above.

Truth definition (7) on page 231 of Montague (1973) has to be replaced by (7*a) and (7*b):

76) *The truth definitions for quantifiers*

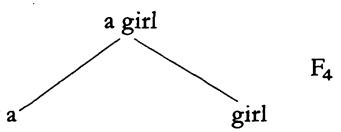
Non-restricted quantification:

(7*a) If $\varphi \varepsilon ME_t$ and u is a variable of type a, then $[Vu\varphi]^{a,i,j,g}$ is 1 iff there exists an x in $D_{a,A,I,J}$ such that $\varphi^{a,i,j,g'}$ is 1, where g′ is the a-assignment like g except for the possible difference that g′(u) is x, and $[Vu\varphi]^{a,i,j,g}$ is 0 iff for every x in $D_{a,A,I,J}$ $\varphi^{a,i,j,g'}$ is 0, where g′ is as described above; and similarily for $\Lambda u\varphi$.

Restricted quantification:

(7*b) If $\varphi, \psi \varepsilon ME_t$ and u is a variable of type a, then $[Vu\exists[\varphi]\psi]^{a,i,j,g}$ is 1 iff there exists an x in $D_{a,A,I,J}$ such that $\varphi^{a,i,j,g'}$ is 1 and $\psi^{a,i,j,g'}$ is 1, where g′ is as described in (7*a); and $[Vu\exists[\varphi]\psi]^{a,i,j,g}$ is 0 iff there exists an x in $D_{a,A,I,J}$ such that $\varphi^{a,i,j,g'}$ is 1 and for every x in $D_{a,A,I,J}$ $\psi^{a,i,j,g'}$ is 0.
If $\varphi, \psi \varepsilon ME_t$ and u is a variable of type a, then $[\Lambda u\exists[\varphi]\psi]^{a,i,j,g}$ is 1 iff for each x in $D_{a,A,I,J}$, if $\varphi^{a,i,j,g'}$ is true, then $\psi^{a,i,j,g'}$ is true and for some x in $D_{a,A,I,J}$ $\varphi^{a,i,j,g'}$ is true; and $[\Lambda u\exists[\varphi]\psi]^{a,i,j,g}$ is 0 iff for some x in $D_{a,A,I,J}$ $\varphi^{a,i,j,g'}$ is 1 and $\psi^{a,i,j,g'}$ is 0, where g′ is as described in (7*a).

It remains to provide more adequate translations for the English quantifiers discussed. Rather than introducing quantifiers by means of a syntactic rule (as in PTQ) let us follow a suggestion by R. Thomason (1972) and define quantifiers in our extension as members of the set $B_{T/CN}$. Thus a term like **a girl** will be analyzed as follows:



The syntactic and translation rules necessary for the combination of quantifiers and common nouns will be called *S4 and *T4, respectively. The original rules S4 and T4 (which serve to combining subjects with verbs to make non-negated present tense sentences) will be made part of the *rules of tense and sign*, S17 and T17, in our extension. The new rules *S4 and *T4 are rules of functional application and have the following form:

*S4 If $\zeta \varepsilon P_{CN}$ and $q \varepsilon P_{T/CN}$, then $F_4(q,\zeta) \varepsilon P_T$ and $F_4(q,\zeta) = q\zeta$.
*T4 If $\zeta \varepsilon P_{CN}$ and $q \varepsilon P_{T/CN}$, and ζ and q translate into ζ′ and q′, respectively, then $F_4(q,\zeta)$ translates into q′ (^ζ′).

We define the following new set of basic expressions:

77) $B_{T/CN} = \{a(n), \text{any}, \text{every}, \text{some}, \text{the}\}$

where

a(n) translates as $\hat{Q}\hat{P}Vx[Q\{x\} \wedge P\{x\}]$
any translates as $\hat{Q}\hat{P}\Lambda x[Q\{x\} \rightarrow P\{x\}]$
every translates as $\hat{Q}\hat{P}\Lambda x\exists[Q\{x\}]P\{x\}$
some translates as $\hat{Q}\hat{P}Vx\exists[Q\{x\}]P\{x\}$
the translates as $\hat{Q}\hat{P}\Lambda x\exists[\Lambda y[Q\{y\} \longleftrightarrow y = x]]P\{x\}$

In accordance with our discussion in 1.3 we treat **every, the,** and **some,** but not the indefinite article and **any,** as existential P-inducers and translate them by means of *restricted quantification.*

Consider now example (78):

78) John has kissed every girl at the party

In the original version of PTQ (78) translates as

79) $H\Lambda x[\text{girl a.t.p.}'(x) \rightarrow \text{kiss}'({}^{\wedge}j, \hat{P}P\{x\})]$

and is *true* with respect to a possible world *i* where no girls a.t.p. exist—a result that is contrary to linguistic intuition. In our extension of PTQ, on the other hand, (78) translates as (80):

80) $H\Lambda x\exists[\text{girl a.t.p.}'(x)]\ \text{kiss}'({}^{\wedge}j, \hat{P}P\{x\})$

(80) is a superior representation of sentence (78) because (80) will be truth-valueless with respect to world *i* where no girls a.t.p. exist.

Note, finally, that in contrast to the original version of PTQ we translate the definite article by means of a universal quantifier rather than an existential quantifier—in accordance with section 1.4 above. Our present translation of singular **the** is still preliminary, however, because it doesn't capture the *definiteness* of **the.** In Hausser (1974b) definiteness is treated as an indexical property. Singular **the,** for example, is translated as

81) $\hat{Q}\hat{P}\Lambda x\exists[\Lambda y[(Q\{y\} \wedge \Gamma\{y\}) \longleftrightarrow y = x]]P\{x\}$

where Γ is a contextual variable. In Hausser (1974b) a procedure is specified for replacing Γ with contextual information (i.e. Γ is replaced by properties derived from propositions).

3.2. *Implementing verbs as P-inducers*

By extending the intensional logic of PTQ to restricted quantification we provided a general way of implementing quantifiers as existential P-inducers. In order to implement verbs like **stop** or **regret** as P-inducers, however, we have

to extend the intensional logic of PTQ further. The predicates of intensional logic corresponding to **stop to** and **regret that** are interpreted as functions from *intensions* of a certain type to *extensions* of another type. The general truth definition for such function-argument expressions as

82) stop′ (date Mary′)

is stated under (4) on page 231 of PTQ (repeated here as (83)).

83) (4) If $\alpha \varepsilon ME_{\langle a,b\rangle}$ and $\beta \varepsilon ME_a$ then $[\alpha(\beta)]^{a,i,j,g}$ is $\alpha^{a,i,j,g}(\beta^{a,i,j,g})$ (that is the value of the function $\alpha^{a,i,j,g}$ for the argument $\beta^{a,i,j,g}$).

What we would like to obtain now is that a sentence like

84) John stopped to date Mary

is # in our extension (with respect to a given interpretation) if the presupposition of (84), namely

85) John used to date Mary.

is not 1 with respect to the interpretation in question. Similarily, in the case of sentence (86):

86) John regrets that the present king of France is bald.

We want (86) to be # with respect to an interpretation if

87) The present king of France is bald

is not 1.

The first step towards this goal is the definition of special truthconditions for the P-inducers **stop to′** and **regret that′**. Let u, v ε $ME_{\langle s,e\rangle}$, $\delta \varepsilon ME_{\langle\langle s,e\rangle,t\rangle}$, and $p \varepsilon ME_{\langle s,t\rangle}$.

88) The value of the function **stop to′** $(u, \hat{v}\delta(v))^{a,i,j,g}$ is # if $H(\delta(u))^{a,i,j,g}$ is not 1.

89) The value of the function **regret that′** $(u, p)^{a,i,j,g}$ is # if $\check{p}^{a,i,j,g}$ is not 1.

Note that we use # to indicate undefined denotations of any type—and not just to indicate an undefined truthvalue! For example, **regret that′** in (89) is a function from $D_{\langle s,t\rangle,A,I,J}$ to $D_{\langle F(IV)\rangle,A,I,J}$. Thus our move to define "truthvalueless" as *undefined truthvalue* rather than a third truthvalue is motivated not only by our desire to avoid the well known objections to many-valued logics, but is an essential part of our overall approach to implementing presuppositions into PTQ. (88) and (89) *restrict the domain* of the functions **stop to′** and **regret that′**.

Adding special truth conditions for P-inducers like **stop′** and **regret that′** (which state the respective presuppositions induced) does not suffice, however. Consider, for example, (86):

86) John regrets that the present king of France is bald

Let us assume that [the present king of France is bald′]^{a,i,j,g} is 0. Consequently, [regret that′ (ˆthe present king of France is bald′)]^{a,i,j,g} is # (because of (89)). The point, however, is that [regret that′ (ˆthe ...)]^{a,i,j,g} is itself *argument* of the function **John′** = $\hat{P}P\{\hat{}j\}$. Note that according to the translation rules of PTQ $\hat{P}P\{\hat{}j\}$ takes the *intension* of [**regret that′** (ˆ**the** ...)], and the intension is defined even if a certain extension, namely [**regret that′** (...)]^{a,i,j,g}, is #. Therefore, in order to insure that $\hat{P}P\{\hat{}j\}$ (ˆ**regret that′** (...))^{a,i,j,g} is # if [**regret that′** (...)]^{a,i,j,g} is #, we have to make the additional assumption that the truthvalue of $\hat{P}P\{\hat{}j\}^{a,i,j,g}$ is defined only if the *extension of its argument is defined*. In other words, we have to add the following condition to truthdefinition (4) of PTQ c.f. (83) above):

90) $\alpha(\beta)^{a,i,j,g}$ is # whenever [ˇβ]^{a,i,j,g} is #, for *most* α!

It would lead to intuitively undesirable results if we implemented (90) for *all* predicates α. Consider for example (91):

91) John believes that Bill stopped to date Mary

The *de dicto* reading of (91) should be bivalent even if the complement sentence "John stopped dating Mary" is # (because the presupposition "John used to date Mary" is not fulfilled). Thus the value of the function [**believe that′**]^{a,i,j,g} *should be defined* for the argument (ˆBill stopped dating Mary′), even if the extension of [**Bill stopped to date Mary′**]^{a,i,j,g} is #. Thus **believe that′** is an exception to (90).

Let us call predicates which are subject to condition (90) *extensional predicates* and the exceptions to (90) *intensional predicates*. Examples of intensional predicates are:

92) **seek′, conceive′** ε $ME_{\langle\langle s, F(T)\rangle, F(IV)\rangle}$
try to′, want to′ ε $ME_{\langle\langle s, F(IV)\rangle, F(IV)\rangle}$
believe that′, assume that′ ε $ME_{\langle\langle s, F(IV)\rangle, F(t)\rangle}$

Based on the distinction between extensional and intensional predicates we can now formulate a suitable replacement of truth definition (4) (c.f. (83) above):

93) (*4) If α ε $ME_{\langle a, b\rangle}$ and β ε ME_a, then $[\alpha(\beta)]^{a,i,j,g}$ is $\alpha^{a,i,j,g}(\beta^{a,i,j,g})$ iff
(i) [ˇβ]^{a,i,j,g} is not # or
(ii) α is an *intensional* predicate.
[Otherwise $\alpha^{a,i,j,g}(\beta^{a,i,j,g})$ is #.]
Furthermore, if u, v ε $ME_{\langle s, e\rangle}$, δ ε $ME_{\langle\langle s, e\rangle, t\rangle}$, p ε $ME_{\langle s, t\rangle}$, and $\mathscr{P}$ ε $ME_{\langle\langle s, \langle\langle s, e\rangle, t\rangle\rangle, t\rangle}$, then
(a) the value of the function **stop to′**$(u, \hat{v}\delta(v))^{a,i,j,g}$ is # if $H(\delta(u))^{a,i,j,g}$ is not 1;
(b) the value of the function **regret that′** $(u, p)^{a,i,j,g}$ is # if $\check{p}^{a,i,j,g}$ is not 1;
(c) the value of the function **look for′** $(u, \mathscr{P})^{a,i,j,g}$ is # if **believe that′** $(u, \mathscr{P}$ (**exist′**))^{a,i,j,g} is not 1.

(*4a) and (*4b) represent our earlier findings regaring the P-inducers **stop** and **regret that.** (*4c) implements the assumption that one cannot look for something unless one believes it to exist. Thus the non-specific reading of

94) John is looking for a unicorn

presupposes in our extension

95) John believes that there exists at least one unicorn

(95) is quite different from (96)

96) There exists at least one unicorn

which is of course *not* presupposed by (94) (or (95)).

The initial introduction of an undefined extension into the interpretation process is caused solely by special P-inducer truthconditions: *7b, *4a, *4b, and *4c. It is the characteristic property of extensional predicates that they "pass along" undefined denotations, i.e. extensional predicates have undefined values if the extension of their argument is undefined. Intensional predicates on the other hand, are not sensitive to a possibly undefined extension of their argument. This means that intensional predicates cancel P-induced presuppositions. Still, intensional predicates are not necessarily *total* functions in our extension. For example, **look for′** is an intensional predicate and thus has defined values even if the extension of its argument is undefined (under the interpretation in question). This is well motivated because (97)

97) John is looking for the present king of France.

should be bivalent on the *de dicto* reading even if the extension of 'the present king of France' is undefined (i.e. no king of France exists at the point of reference in question). Nevertheless, (97) will be # in our extension *if John does not believe* that the present king of France exists. The reason is the P-inducer-condition (*4c). Thus **look for′** is defined here as an intensional predicate which may have undefined values.

We have now completed the implementation of presuppositions into PTQ by means of partial functions. It is a matter of further linguistic research to extend the present set of P-inducers, comprising **the, some, every, stop, regret that,** and **look for.** The formal implementation of additional P-inducers into PTQ will be routine, however. We have, furthermore, implemented two different canceling contexts: the logical connectives and intensional predicates like **seek′, look for′, try to′, believe that′,** etc. It seems to me that the study of further canceling contexts could provide important insights for a systematic model-semantic analysis of natural language.

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